Notation and Symbols

Types of Numbers

Natural Numbers (Counting Numbers): $\mathbb{N} = \{1, 2, 3, 4, 5, 6, ...\}$

Whole Numbers: $\mathbb{W} = \{0, 1, 2, 3, 4, 5, 6, ...\}$

Integers: $\mathbb{Z} = \{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$

Rational Numbers: A rational number is a number that can be written in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Irrational Numbers: An irrational number is a number that can be written as an infinite nonrepeating decimal.

Real Numbers: The real numbers consist of all rational and irrational numbers.

Complex Number: The complex numbers consist of all real numbers and the even roots of negative numbers.

Equality and Inequality Symbols

$=$	"is equal to"	$<$	"is less than"	$\leq$	"is less than or equal to"
$\neq$	"is not equal to"	$>$	"is greater than"	$\geq$	"is greater than or equal to"

Absolute Value

The **absolute value** of a real number is its distance from 0. Symbolically,

$|a| = a$ if a is a positive number or 0.

$|a| = -a$ if a is a negative number.

Principles and Properties

Properties of Addition and Multiplication		
For Addition	**Name of Property**	**For Multiplication**
$a + b = b + a$	**Commutative Property**	$ab = ba$
$(a + b) + c = a + (b + c)$	**Associative Property**	$a(bc) = (ab)c$
$a + 0 = 0 + a = a$	**Identity**	$a \cdot 1 = 1 \cdot a = a$
$a + (-a) = 0$	**Inverse**	$a \cdot \frac{1}{a} = 1 \ (a \neq 0)$
Zero-Factor Law: $a \cdot 0 = 0 \cdot a = 0$		**Distributive Property:** $a(b + c) = a \cdot b + a \cdot c$

Addition Principle of Equality

$A = B$, $A + C = B + C$, and $A - C = B - C$ have the same solutions (where A, B, and C are algebraic expressions).

Multiplication (or Division) Principle of Equality

$A = B$, $AC = BC$, and $\frac{A}{C} = \frac{B}{C}$ have the same solutions (where A and B are algebraic expressions and C is any nonzero constant, $C \neq 0$).

Properties of Radicals

If a and b are positive real numbers, n is a positive integer, m is any integer, and $\sqrt[n]{a}$ is a real number then

1. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ 2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3. $\sqrt[n]{a} = a^{\frac{1}{n}}$
4. $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$

or, in radical notation, $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

Rules

Divisibility Rules

For 2: If the last digit (units digit) of an integer is 0, 2, 4, 6, or 8, then the integer is divisible by 2.

For 3: If the sum of the digits of an integer is divisible by 3, then the integer is divisible by 3.

For 5: If the last digit of an integer is 0 or 5, then the integer is divisible by 5.

For 6: If the integer is divisible by both 2 and 3, then it is divisible by 6.

For 9: If the sum of the digits of an integer is divisible by 9, then the integer is divisible by 9.

For 10: If the last digit of an integer is 0, then the integer is divisible by 10.

Rules for Exponents

For nonzero real numbers a and b and integers m and n:

The exponent 1: $a = a^1$

The exponent 0: $a^0 = 1$

The product rule: $a^m \cdot a^n = a^{m+n}$

The quotient rule: $\dfrac{a^m}{a^n} = a^{m-n}$

Negative exponents: $a^{-n} = \dfrac{1}{a^n}$

Power rule: $\left(a^m\right)^n = a^{mn}$

Power of a product: $(ab)^n = a^n b^n$

Power of a quotient: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Linear Equations

Summary of Formulas and Properties of Lines

Standard Form:

$Ax + By = C$ where A and B do not both equal 0

Slope of a line:

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$

Slope-intercept form:

$y = mx + b$ with slope m and y-intercept $(0,b)$

Point-slope form:

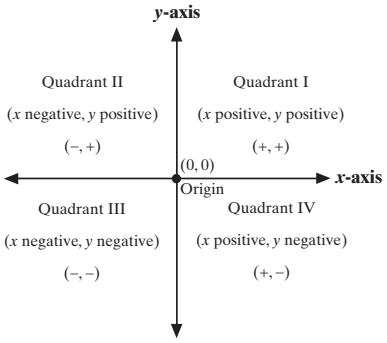
$y - y_1 = m(x - x_1)$ with slope m and point (x_1, y_1) on the line

Horizontal line, slope 0: $y = b$

Vertical line, undefined slope: $x = a$

1. **Parallel lines** have the same slope.
2. **Perpendicular lines** have slopes that are negative reciprocals of each other.

Cartesian Coordinate System



Factoring Polynomials

Special Factoring Techniques

1. $x^2 - a^2 = (x + a)(x - a)$: difference of two squares
2. $x^2 + 2ax + a^2 = (x + a)^2$: square of a binomial sum
3. $x^2 - 2ax + a^2 = (x - a)^2$: square of a binomial difference
4. $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$: sum of two cubes
5. $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$: difference of two cubes

Inequalities

Linear Inequalities

Linear Inequalities have the following forms where a, b, and c are real numbers and $a \neq 0$:

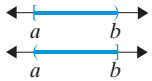
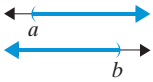
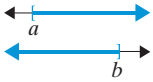
$ax + b < c$ and $ax + b \leq c$

$ax + b > c$ and $ax + b \geq c$

Compound Inequalities

The inequalities $c < ax + b < d$ and $c \leq ax + b \leq d$ are called **compound linear inequalities**. (This includes $c < ax + b \leq d$ and $c \leq ax + b < d$ as well.)

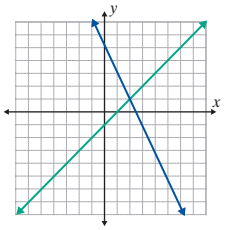
Interval Notation

Type of Interval	Algebraic Notation	Interval Notation	Graph
Open Interval	$a < x < b$	(a, b)	
Closed Interval	$a \leq x \leq b$	$[a, b]$	
Half-open Interval	$a \leq x < b$ $a < x \leq b$	$[a, b)$ $(a, b]$	
Open Interval	$x > a$ $x < b$	(a, ∞) $(-\infty, b)$	
Half-open Interval	$x \geq a$ $x \leq b$	$[a, \infty)$ $(-\infty, b]$	

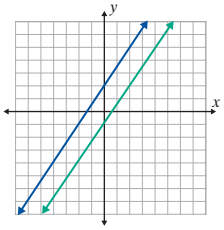
Systems of Linear Equations

Systems of Linear Equations (Two Variables)

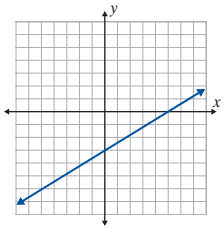
Consistent
(One solution)



Inconsistent
(No solution)



Dependent
(Infinite number of solutions)

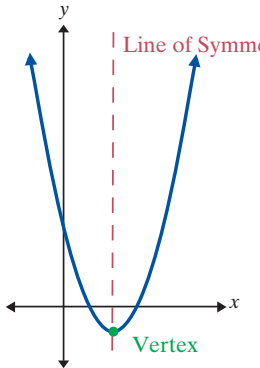


Quadratic Equations

Quadratic Formula

The solutions of the general quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

General Information on Quadratic Functions



For the quadratic function $y = ax^2 + bx + c$

1. If $a > 0$, the parabola "opens upward."
2. If $a < 0$, the parabola "opens downward."
3. $x = -\dfrac{b}{2a}$ is the line of symmetry.
4. The vertex (turning point) occurs where $x = -\dfrac{b}{2a}$.

   Substitute this value for x in the function to find the y-value of the vertex. The vertex is the lowest point on the curve if the parabola opens upward and it is the highest point on the curve if the parabola opens downward.

Relationships Between Measurements in the Metric System

Length

1 millimeter (mm) = 0.001 meter 1 m = 1000 mm
1 centimeter (cm) = 0.01 meter 1 m = 100 cm
1 decimeter (dm) = 0.1 meter 1 m = 10 dm
1 meter (m) = 1.0 meter
1 dekameter (dam) = 10 meters
1 hectometer (hm) = 100 meters
1 kilometer (km) = 1000 meters

Mass

1 milligram (mg) = 0.001 gram 1 g = 1000 mg
1 centigram (cg) = 0.01 gram
1 decigram (dg) = 0.1 gram
1 gram (g) = 1.0 gram
1 dekagram (dag) = 10 grams
1 hectogram (hg) = 100 grams
1 kilogram (kg) = 1000 grams 1 g = 0.001 kg
1 metric ton (t) = 1000 kilograms 1 kg = 0.001 t
1t = 1000 kg = 1,000,000 g = 1,000,000,000 mg

Area

Small Area	Land Area
$1\ cm^2 = 100\ mm^2$ $1\ dm^2 = 100\ cm^2 = 10\,000\ mm^2$ $1\ m^2 = 100\ dm^2 = 10\,000\ cm^2 = 1\,000\,000\ mm^2$	$1\ a = 100\ m^2$ $1\ ha = 100\ a = 10\,000\ m^2$

Volume

$1\ cm^3 = 1000\ mm^3$
$1\ dm^3 = 1000\ cm^3 = 1\,000\,000\ mm^3$
$1\ m^3 = 1000\ dm^3 = 1\,000\,000\ cm^3 = 1\,000\,000\,000\ mm^3$

Liquid Volume

1 milliliter (mL) = 0.001 liter = $1\ cm^3$
1 liter (L) = 1.0 liter = $1\ dm^3$ = 1000 mL
1 hectoliter (hL) = 100 liters
1 kiloliter (kL) = 1000 liters = 10 hL = $1\ m^3$

U.S. Customary and Metric Equivalents

U.S. to Metric	Metric to U.S.
Length	
1 in. = 2.54 cm (exact) 1 ft = 0.305 m 1 yd = 0.914 m 1 mi = 1.61 km	1 cm = 0.394 in. 1 m = 3.28 ft 1 m = 1.09 yd 1 km = 0.62 mi
Area	
$1\ in.^2 = 6.45\ cm^2$ $1\ ft^2 = 0.093\ m^2$ $1\ yd^2 = 0.836\ m^2$ 1 acre = 0.405 ha	$1\ cm^2 = 0.155\ in.^2$ $1\ m^2 = 10.764\ ft^2$ $1\ m^2 = 1.196\ yd^2$ 1 ha = 2.47 acres
Volume	
$1\ in.^3 = 16.387\ cm^3$ $1\ ft^3 = 0.028\ m^3$ 1 qt = 0.946 L 1 gal = 3.785 L	$1\ cm^3 = 0.06\ in.^3$ $1\ m^3 = 35.315\ ft^3$ 1 L = 1.06 qt 1 L = 0.264 gal
Mass	
1 oz = 28.35 g 1 lb = 0.454 kg	1 g = 0.035 oz 1 kg = 2.205 lb

Temperature Equivalents

Celsius		Fahrenheit
100°	Water boils	212°
90°		194°
80°		176°
70°		158°
60°		140°
50°		122°
40°		104°
30°	Comfort	86°
20°	range	68°
10°		50°
0°	Water freezes	32°

Relationships Between Measurements in the U.S. Customary System

Length	Area	Liquid Volume
12 inches (in.) = 1 foot (ft) 36 inches = 1 yard (yd) 3 feet = 1 yard (yd) 5280 feet = 1 mile (mi)	$1\ ft^2 = 144\ in.^2$ $1\ yd^2 = 9\ ft^2$ 1 acre = 4840 yd^2 = 43,560 ft^2	1 pint (pt) = 16 fluid ounces (fl oz) 1 quart (qt) = 2 pt = 32 fl oz 1 gallon (gal) = 4 qt

Formulas and Definitions

Terms Related to the Basic Equation $R \cdot B = A$

R = **rate** or percent (as a decimal or fraction)
B = **base** (number we are finding the percent of)
A = **amount** (a part of the base)

Terms Related to Profit

Profit: The difference between selling price and cost.
profit = selling price – cost

Percent of Profit:

1. Percent of profit **based on cost**: $\dfrac{\text{profit}}{\text{cost}}$
2. Percent of profit **based on selling price**: $\dfrac{\text{profit}}{\text{selling price}}$

Formulas for Calculating Interest

Simple Interest

$$I = P \cdot r \cdot t,$$

Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

where
I = interest (earned or paid)
P = principal (the amount invested or borrowed)
r = annual rate of interest in decimal form
t = time (in years or fraction of a year)
A = the future value of the investment
n = the number of compounding periods in 1 year

Formulas for Calculating Inflation and Depreciation

Inflation

$$A = P(1+r)^t$$

Depreciation

$$V = P(1-r)^t$$

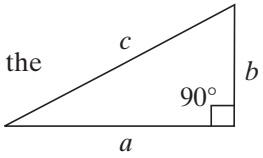
where
P = the original value
r = the annual rate of depreciation or appreciation/inflation in decimal form
t = time (in years or fraction of a year)

List of Common Formulas

Formula	Meaning
$C = \frac{5}{9}(F-32)$	**Temperature** in degrees Celsius C equals $\frac{5}{9}$ times the difference between the Fahrenheit temperature F and 32.
$d = rt$	The **distance traveled** d equals the product of the rate of speed r and the time t.
$L = 2\pi rh$	The **lateral surface area** L (top and bottom not included) of a cylinder is equal to 2π times the radius r of the base times the height h.
$F = ma$	In physics, the **force** F acting on an object is equal to its mass m times acceleration a.

The Pythagorean Theorem

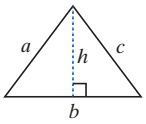
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs: $\boldsymbol{c^2 = a^2 + b^2}$



Geometry

Perimeter and Area

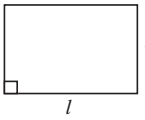
Triangle



Perimeter: $P = a + b + c$

Area: $A = \frac{1}{2}bh$

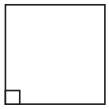
Rectangle



Perimeter: $P = 2l + 2w$

Area: $A = lw$

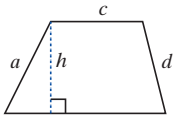
Square



Perimeter: $P = 4s$

Area: $A = s^2$

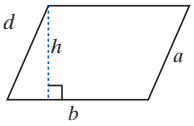
Trapezoid



Perimeter: $P = a + b + c + d$

Area: $A = \frac{1}{2}h(b+c)$

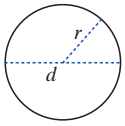
Parallelogram



Perimeter: $P = 2a + 2b$

Area: $A = bh$

Circle

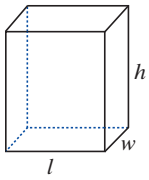


Circumference: $C = 2\pi r = \pi d$

Area: $A = \pi r^2$

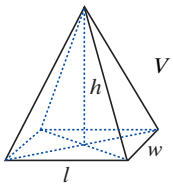
Volume

Rectangular solid



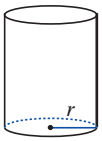
$V = lwh$

Rectangular pyramid



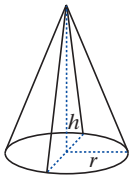
$V = \frac{1}{3}lwh$

Right circular cylinder



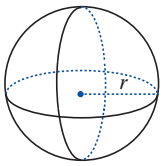
$V = \pi r^2 h$

Right circular cone



$V = \frac{1}{3}\pi r^2 h$

Sphere

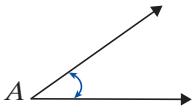


$V = \frac{4}{3}\pi r^3$

Angles Classified by Measure

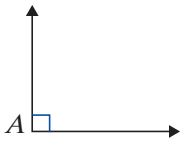
1. Acute

$0° < m\angle A < 90°$



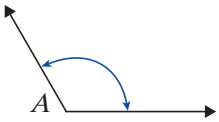
2. Right

$m\angle A = 90°$



3. Obtuse

$90° < m\angle A < 180°$

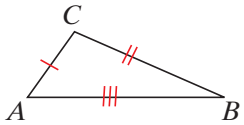


4. Straight

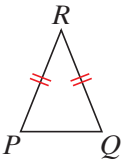
$m\angle A = 180°$

Triangles Classified by Sides

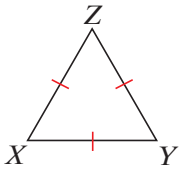
1. Scalene No two sides are equal.



2. Isosceles At least two sides are equal.

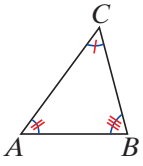


3. Equilateral All three sides are equal.

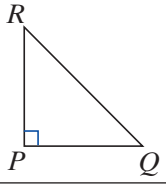


Triangles Classified by Angles

1. Acute All three angles are acute.



2. Right One angle is a right angle.



3. Obtuse One angle is obtuse.