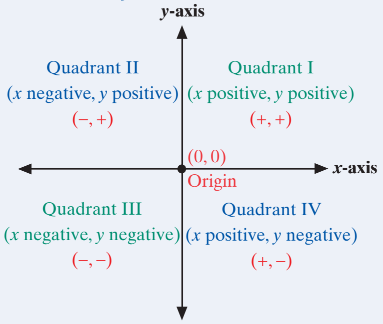
CHAPTER 2 Linear Equations and Functions

Cartesian Coordinate System:



Summary of Formulas and Properties of Straight Lines:

1. $Ax + By = C$ where A and B do not both equal 0. Standard form
2. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$. Slope of a line
3. $y = mx + b$ Slope-intercept form (with slope m, and y-intercept $(0, b)$)
4. $y - y_1 = m(x - x_1)$ Point-slope form
5. $y = b$ Horizontal line, slope 0
6. $x = a$ Vertical line, undefined slope
7. Parallel lines have the same slope.
8. Perpendicular lines have slopes that are negative reciprocals of each other.

Relation, Domain, and Range:

Relation: A **relation** is a set of ordered pairs of real numbers.

Domain: The **domain**, ***D***, of a relation is the set of all first coordinates in the relation.

Range: The **range**, ***R***, of a relation is the set of all second coordinates in the relation.

Function:

A **function** is a relation in which each domain element has exactly one corresponding range element.

Vertical Line Test:

If **any** vertical line intersects the graph of a relation at more than one point, then the relation graphed is **not** a function.

Linear Inequality Terminology:

Half-plane: A straight line separates a plane into two **half-planes**.

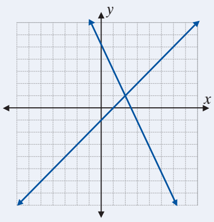
Boundary line: The line itself is called the **boundary line**.

Closed half-plane: If the boundary line is included, then the half-plane is said to be **closed**.

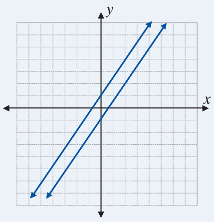
Open half-plane: If the boundary line is not included, then the half-plane is said to be **open**.

CHAPTER 3 Systems of Linear Equations

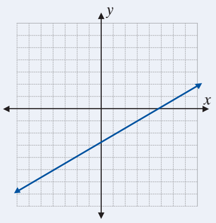
Systems of Linear Equations (Two Variables):



Consistent
(One solution)



Inconsistent
(No solution)



Dependent
(Infinite number of solutions)

Matrices:

System of Linear Equations

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

Coefficient Matrix

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

Augmented Matrix

$$\left[\begin{array}{cc:c} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{array}\right]$$

Elementary Row Operations:

1. Interchange two rows.
2. Multiply a row by a nonzero constant.
3. Add a multiple of a row to another row.

Upper Triangular Form and Row Echelon Form:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

A matrix is in **upper triangular form** if its entries in the lower left triangular region are all 0's. If a_{11}, a_{22}, and a_{33} (the entries along the main diagonal) all equal 1 when the matrix is in upper triangular form, then the matrix is also in **row echelon form** (or **ref**).

Gaussian Elimination:

1. Write the augmented matrix for the system.
2. Use elementary row operations to transform the matrix into row echelon form.
3. Solve the corresponding system of equations by using back substitution.

Determinants:

Value of a 2 × 2 Determinant:

For the square matrix, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $\det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$.

Value of a 3 × 3 Determinant:

For the square matrix, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $\det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$= a_{11}(\text{minor of } a_{11}) - a_{12}(\text{minor of } a_{12}) + a_{13}(\text{minor of } a_{13})$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}.$$

Cramer's Rule:

For the system $\begin{cases} a_{11}x + a_{12}y = k_1 \\ a_{21}x + a_{22}y = k_2 \end{cases}$

where $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$, $D_x = \begin{vmatrix} k_1 & a_{12} \\ k_2 & a_{22} \end{vmatrix}$, and $D_y = \begin{vmatrix} a_{11} & k_1 \\ a_{21} & k_2 \end{vmatrix}$,

if $D \neq 0$, then $x = \dfrac{D_x}{D}$ and $y = \dfrac{D_y}{D}$ is the unique solution to the system.

CHAPTER 4 Exponents and Polynomials

Properties of Exponents:

For nonzero real numbers a and b and integers m and n,

The Exponent 1: $a = a^1$ (a is any real number.)

The Exponent 0: $a^0 = 1 \quad (a \neq 0)$

Product Rule: $a^m \cdot a^n = a^{m+n}$

Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

Negative Exponents: $a^{-n} = \frac{1}{a^n}$

Power Rule: $\left(a^m\right)^n = a^{mn}$

Power Rule for Products: $(ab)^n = a^n b^n$

Power Rule for Fractions: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Scientific Notation:

$N = a \times 10^n$ where N is a decimal number, $1 \leq a < 10$, and n is an integer.

Classification of Polynomials:

Monomial: polynomial with one term

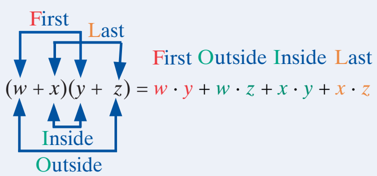
Binomial: polynomial with two terms

Trinomial: polynomial with three terms

Degree: The **degree of a term** is the sum of the exponents of its variables. The **degree of a polynomial** is the largest of the degrees of its terms.

Leading Coefficient: The coefficient of the term with the largest degree.

FOIL Method:



Division Algorithm:

For polynomials P and D, $\frac{P}{D} = Q + \frac{R}{D}$, $(D \neq 0)$ where Q and R are polynomials and the degree of $R <$ the degree of D.

Factoring out the GCF:

1. Find the variable(s) of highest degree and the largest integer coefficient that is a factor of each term of the polynomial. (This is one factor.)
2. Divide this monomial factor into each term of the polynomial resulting in another polynomial factor.

Special Products of Polynomials:

1. $(x+a)(x-a) = x^2 - a^2$: Difference of two squares
2. $(x+a)^2 = x^2 + 2ax + a^2$: Square of a binomial sum
3. $(x-a)^2 = x^2 - 2ax + a^2$: Square of a binomial difference
4. $(x-a)\left(x^2 + ax + a^2\right) = x^3 - a^3$: Difference of two cubes
5. $(x+a)\left(x^2 - ax + a^2\right) = x^3 + a^3$: Sum of two cubes

Quadratic Equation:

An equation that can be written in the form $ax^2 + bx + c = 0$ where a, b, and c are constants and $a \neq 0$.

Zero-Factor Property:

If a and b are real numbers, and $a \cdot b = 0$, then $a = 0$ or $b = 0$ or both.

Factor Theorem:

If $x = c$ is a root of a polynomial equation in the form $P(x) = 0$, then $x - c$ is a factor of the polynomial $P(x)$.

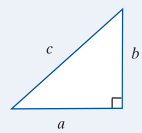
Consecutive Integers:

$n, n+1, n+2, \ldots$

The Pythagorean Theorem:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

$$c^2 = a^2 + b^2$$



CHAPTER 5 Rational Expressions and Rational Equations

Rational Expression:

A **rational expression** is an expression of the form $\frac{P}{Q}$ where P and Q are polynomials and $Q \neq 0$.

Fundamental Principle of Rational Expressions:

If $\frac{P}{Q}$ is a rational expression where $Q \neq 0$ and K is a polynomial where $K \neq 0$, then $\frac{P}{Q} = \frac{P \cdot K}{Q \cdot K}$.

Opposites in Rational Expressions:

For a polynomial P, $\frac{-P}{P} = -1$ where $P \neq 0$.

In particular, $\frac{a-x}{x-a} = \frac{-(x-a)}{x-a} = -1$ where $x \neq a$.

Multiplication with Rational Expressions:

If P, Q, R, and S are polynomials and $Q, S \neq 0$, then $\frac{P}{Q} \cdot \frac{R}{S} = \frac{P \cdot R}{Q \cdot S}$.

Division with Rational Expressions:

If P, Q, R, and S are polynomials and $Q, R, S \neq 0$, then $\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R}$.

Addition and Subtraction with Rational Expressions:

$\frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}$ and $\frac{P}{Q} - \frac{R}{Q} = \frac{P-R}{Q}$ where $Q \neq 0$.

Negative Signs in Rational Expressions:

$-\frac{P}{Q} = \frac{P}{-Q} = \frac{-P}{Q}$

Work Problems:

To solve this type of problem, represent what part of the work is done in one unit of time.

Distance-Rate-Time Problems:

Use the formula $d = rt$, where d = the distance traveled, r = the rate, and t = the time taken, to solve this type of problem.

Variation:

Direct Variation: A variable quantity y varies directly as a variable x if there is a constant k such that $\frac{y}{x} = k$ or $y = kx$.

Inverse Variation: A variable quantity y varies inversely as a variable x if there is a constant k such that $x \cdot y = k$ or $y = \frac{k}{x}$.

CHAPTER 6 Roots, Radicals, and Complex Numbers

Square Roots:

If $b^2 = a$, then b is called a **square root** of a $(a \ge 0)$.

If x is a real number, then $\sqrt{x^2} = |x|$. However, if $x \ge 0$, then we can write $\sqrt{x^2} = x$.

Properties of Square Roots:

If a and b are positive real numbers, then

1. $\sqrt{ab} = \sqrt{a}\sqrt{b}$
2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

For any nonnegative real number x and positive integer m,

3. $\sqrt{x^{2m}} = x^m$
4. $\sqrt{x^{2m+1}} = x^m\sqrt{x}$

Cube Roots:

If $b^3 = a$, then b is called the **cube root** of a. We write $\sqrt[3]{a} = b$.

Properties of Radicals:

1. If n is a positive integer and $b^n = a$, then $b = \sqrt[n]{a} = a^{\frac{1}{n}}$.
2. $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(a^m\right)^{\frac{1}{n}}$ or, in radical notation, $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

To Rationalize a Denominator Containing a Sum or Difference Involving Square Roots:

Rationalize the denominator by multiplying both the numerator and the denominator by the **conjugate of the denominator**.

1. If the denominator is of the form $a - b$, multiply both the numerator and denominator by $a + b$.
2. If the denominator is of the form $a + b$, multiply both the numerator and denominator by $a - b$.

The new denominator will be the difference of two squares and therefore not contain a radical term.

Definition of *i*:

$i = \sqrt{-1}$ and $i^2 = \left(\sqrt{-1}\right)^2 = -1$

If a is positive real number, $\sqrt{-a} = \sqrt{a}\cdot\sqrt{-1} = \sqrt{a}i = i\sqrt{a}$.

Complex Numbers:

The **standard form of a complex number** is $a + bi$ where a and b are real numbers. a is called the **real part** and b is called the **imaginary part**.

CHAPTER 7 Quadratic Equations and Quadratic Functions

Square Root Property:

If $x^2 = c$, then $x = \pm\sqrt{c}$.

If $(x-a)^2 = c$, then $x - a = \pm\sqrt{c}$ $\left(\text{or } x = a \pm \sqrt{c}\right)$.

Completing the Square:

To complete the square, find the third term of a perfect square trinomial when the first two terms are given. The trinomial should have the following characteristics:

1. The leading coefficient (the coefficient of x^2) is 1.
2. The constant term is the square of $\frac{1}{2}$ of the coefficient of x.

Quadratic Formula:

For the quadratic equation $ax^2 + bx + c = 0$, where $a \ne 0$, the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Discriminant:

The expression $b^2 - 4ac$, the part of the quadratic formula that lies under the radical sign, is called the **discriminant**.

If $b^2 - 4ac > 0 \rightarrow$ There are two real solutions.

If $b^2 - 4ac = 0 \rightarrow$ There is one real solution, $x = -\frac{b}{2a}$.

If $b^2 - 4ac < 0 \rightarrow$ There are two nonreal solutions.

Projectiles:

$h = -16t^2 + v_0 t + h_0$, where h is the height of the object in feet, t is the time object is in the air in seconds, v_0 is the beginning velocity in feet per second, and h_0 is the beginning height in feet.

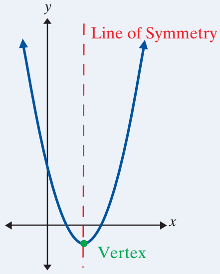
Parabolas:

Parabolas of the form $y = ax^2 + bx + c$:

If $a > 0$, the parabola opens upward.
If $a < 0$, the parabola opens downward.
The bigger $|a|$ is, the narrower the opening.
The smaller $|a|$ is, the wider the opening.

Vertex: $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$

Line of Symmetry: $x = -\frac{b}{2a}$



Parabolas of the form $y = a(x - h)^2 + k$:

Vertex: (h, k)
Line of Symmetry: $x = h$
The graph is a horizontal shift of h units and a vertical shift of k units of the graph of $y = ax^2$.

Minimum and Maximum Values:

For a parabola with its equation given in the form $y = a(x-h)^2 + k$:

1. If $a > 0$, then the parabola opens upward, (h, k) is the lowest point, and $y = k$ is called the **minimum value** of the function.
2. If $a < 0$, then the parabola opens downward, (h, k) is the highest point, and $y = k$ is called the **maximum value** of the function.

CHAPTER 8 Exponential and Logarithmic Functions

Algebraic Operations with Functions:

For functions $f(x)$ and $g(x)$ where x is in the domain of both functions,

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Composite Functions:

For functions $f(x)$ and $g(x)$, $(f \circ g)(x) = f(g(x))$.

Domain of $f \circ g$: The domain of $f \circ g$ consists of those values of x in the domain of g for which $g(x)$ is in the domain of f.

One-to-One Functions:

A function is a **one-to-one function** (or **1–1 function**) if for each value of y in the range there is only one corresponding value of x in the domain.

Horizontal Line Test:

A function is 1–1 if no horizontal line intersects the graph of the function at more than one point.

Inverse Functions:

If f is a 1–1 function with ordered pairs of the form (x, y), then its **inverse function**, denoted f^{-1}, is also a 1–1 function with ordered pairs of the form (y, x).

If f and g are 1–1 functions and $f(g(x)) = x$ for all x in D_g and $g(f(x)) = x$ for all x in D_f, then f and g are **inverse functions**.

Exponential Functions:

An **exponential function** is a function of the form $f(x) = b^x$ where $b > 0$, $b \neq 1$, and x is any real number.

Concepts of Exponential Functions:

For $b > 1$:

1. $b^x > 0$
2. b^x increases to the right and is called an **exponential growth function**
3. $b^0 = 1$, so $(0, 1)$ is on the graph
4. b^x approaches the x-axis for negative values of x (The x-axis is a horizontal asymptote.)

For $0 < b < 1$:

1. $b^x > 0$
2. b^x decreases to the right and is called an **exponential decay function**
3. $b^0 = 1$, so $(0, 1)$ is on the graph
4. b^x approaches the x-axis for positive values of x (The x-axis is a horizontal asymptote.)

Compound Interest:

Compound interest on a principal P invested at an annual interest rate r (in decimal form) for t years that is compounded n times per year can be calculated using the following formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A is the amount accumulated.

The Number *e*:

The number e is defined to be $e = 2.718281828459\ldots$

Continuously Compounded Interest:

Continuously compounded interest on a principal P invested at an annual interest rate r for t years, can be calculated using the following formula:

$$A = Pe^{rt}$$

where A is the amount accumulated.

Logarithms:

For $b > 0$ and $b \neq 1$, $x = b^y$ is equivalent to $y = \log_b x$.

Properties of Logarithms:

For $b, x, y > 0$, $b \neq 1$, and any real number r:

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $x = b^{\log_b x}$
4. $\log_b b^x = x$
5. $\log_b (xy) = \log_b x + \log_b y$
6. $\log_b \frac{x}{y} = \log_b x - \log_b y$
7. $\log_b x^r = r \cdot \log_b x$

Properties of Equations with Exponents and Logarithms:

For $b > 0$ and $b \neq 1$:

1. If $b^x = b^y$, then $x = y$.
2. If $x = y$, then $b^x = b^y$.
3. If $\log_b x = \log_b y$, then $x = y$ $(x > 0$ and $y > 0)$.
4. If $x = y$, then $\log_b x = \log_b y$ $(x > 0$ and $y > 0)$.

Change of Base:

For $a, b, x > 0$ and $a, b \neq 1$, $\log_b x = \frac{\log_a x}{\log_a b}$.

CHAPTER 9 Conic Sections

Horizontal and Vertical Translations:

Given a graph $y = f(x)$, the graph of $y = f(x - h) + k$ is:

1. a horizontal translation of $f(x)$ by h units and
2. a vertical translation of $f(x)$ by k units.

Horizontal Parabolas:

Parabolas of the form $x = ay^2 + by + c$ or $x = a(y-k)^2 + h$

If $a > 0$, the parabola opens right.

If $a < 0$, the parabola opens left.

Vertex: (h, k)

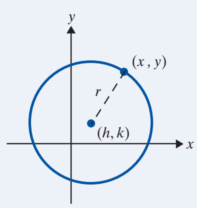
Line of Symmetry: $y = k$

Distance Formula (distance between two points):

For two points, $P(x_1, y_1)$ and $Q(x_2, y_2)$, in a plane, the distance between the points is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Circles:

A **circle** is the set of all points in a plane that are a fixed distance from a fixed point.



The fixed point is called the **center** of a circle.

The distance from the center to any point on the circle is called the **radius** of the circle.

The distance from one point on the circle to another point on the circle measured through the center is the **diameter** of the circle.

Standard form of a circle: $(x - h)^2 + (y - k)^2 = r^2$

The radius is r and the center is at (h, k).