4.1 Translating English Phrases and Algebraic Expressions

Translating English Phrases into Algebraic Expressions

Algebra is a language of mathematicians, and to understand mathematics, you must understand the language. We want to be able to change English phrases into their "algebraic" equivalents and vice versa. So if a problem is stated in English, we can translate the phrases into algebraic symbols and proceed to solve the problem according to the rules and methods developed for algebra.

Certain words are the keys to the basic operations. Some of these words are listed here and highlighted in boldface in Example 1.

Key Words To Look For When Translating Phrases				
Addition	**Subtraction**	**Multiplication**	**Division**	**Exponent (Powers)**
add sum plus more than increased by	subtract (from) difference minus less than decreased by less	multiply product times twice of (with fractions and percent)	divide quotient	square of cube of

The following examples illustrate how these key words used in English phrases can be translated into algebraic expressions. **Note that in each case "a number" or "the number" implies the use of a variable (an unknown quantity).**

Concept Check:

Determine the key word (or phrase) that does not translate to the given operation.

1. Multiplication

   of | twice | times | squared | product

2. Subtraction

   more than | decreased by | difference | less | minus

Example 1: Translating English Phrases into Algebraic Expressions

Change each phrase into an equivalent algebraic expression.

	English Phrase	Algebraic Expression
a.	the **product** of 3 and x 3 **times** x 3 **multiplied by** the number represented by x	$3x$
b.	3 **added to** a number the **sum of** z and 3 z **plus** 3 3 **more than** z z **increased by** 3	$z+3$
c.	**twice** the **sum** of x and 1 the **product** of 2 with the **sum** of x and 1 2 **times** the quantity found by **adding** a number to 1	$2(x+1)$
d.	**twice** x **plus** 1 the **sum** of **twice** x and 1 2 **times** x **increased by** 1 1 **more than** the **product** of 2 and a number	$2x+1$
e.	the **difference** between 5 **times** a number and 3 3 **less than** the **product** of a number and 5 5 **times** a number **minus** 3 3 **subtracted from** $5n$ 5 **multiplied by** a number, **less** 3	$5n-3$
f.	the **square of** a number a number **squared**	x^2
g.	the **cube of** a number a number **cubed**	n^3

Notes

In Example 1b, the phrase "the sum of z and 3" was translated as $z + 3$. If the expression had been translated as $3 + z$, there would have been no mathematical error because addition is commutative. That is, $z + 3 = 3 + z$. However, in part e, the phrase " 3 less than the product of a number and 5" must be translated as it was because subtraction is **not** commutative. Thus

" 3 less than 5 times a number" means $5n - 3$

while " 5 times a number less than 3" means $3 - 5n$

and " 3 less 5 times a number" means $3 - 5n$.

Therefore, be very careful when writing and/or interpreting expressions indicating subtraction. Be sure that the subtraction is in the order indicated by the wording in the problem. The same is true with expressions involving division.

The words **quotient** and **difference** deserve special mention because their use implies that the numbers given are to be operated on in the order given. That is, division and subtraction are done with the values in the same order that they are given in the problem. For example:

the quotient of y and 5 $\rightarrow \frac{y}{5}$

the quotient of 5 and y $\rightarrow \frac{5}{y}$

the difference between 6 and x $\rightarrow 6 - x$

the difference between x and 6 $\rightarrow x - 6$

If we did not have these agreements concerning subtraction and division, then the phrases just illustrated might have more than one interpretation and be considered **ambiguous**.

An **ambiguous phrase** is one whose meaning is not clear or for which there may be two or more interpretations. This is a common occurrence in ordinary everyday language, and misunderstandings occur frequently. Imagine the difficulties diplomats have in communicating ideas from one language to another trying to avoid ambiguities. Even the order of subjects, verbs, and adjectives may not be the same from one language to another. Translating grammatical phrases in any language into mathematical expressions is quite similar. To avoid ambiguous phrases in mathematics, we try to be precise in the use of terminology, to be careful with grammatical construction, and to follow the rules for order of operations.

Translating Algebraic Expressions into English Phrases

Consider the three expressions to be translated into English:

$$7\,(n+1),\ 6\,(n-3),\ \text{and}\ 7n+1.$$

In the first two expressions, we indicate the parentheses with a phrase such as "the quantity" or "the sum of"" or "the difference between." **Without the parentheses, we agree that the operations used in the expression are to be indicated in the order given.** Thus

$7\,(n+1)$ can be translated as "seven times the sum of a number and 1,"

$6\,(n-3)$ can be translated as "six times the difference between a number and 3,"

while $7n+1$ can be translated as "seven times a number plus 1."

Example 2: Translating Algebraic Expressions to English Phrases

Write an English phrase that indicates the meaning of each algebraic expression.

	Algebraic Expression	Possible English Phrase
a.	$5x$	the product of 5 and a number
b.	$2n+8$	twice a number increased by 8
c.	$3\,(a-2)$	three times the difference between a number and 2

Example 3: Translating English Phrases to Algebraic Expressions

Change each phrase into an equivalent algebraic expression.

	English Phrase	Algebraic Expression
a.	the quotient of a number and -4	$\dfrac{x}{-4}$
b.	6 less than 5 times a number	$5y-6$
c.	twice the sum of 3 and a number	$2\,(3+n)$
d.	the number of minutes in h hours	$60\ h$
e.	the cost of renting a truck for one day and driving x miles if the rate is \$30 per day plus \$0.25 per mile	$30+0.25x$

Exercises 4.1

Translate each algebraic expression into an equivalent English phrase. There may be more than one correct translation.

1. $4x$
2. $-9x$
3. $x+5$
4. $4x-7$
5. $7(x+1.1)$
6. $3.2(x+2.5)$
7. $-2(x-8)$
8. $10(x+4)$
9. $\dfrac{6}{(x-1)}$
10. $\dfrac{9}{(x+3)}$
11. $5(2x+3)$
12. $3(4x-5)$

Write each pair of expressions in words. Notice the differences between the algebraic expressions and the corresponding English phrases.

13. $3x+7;\ 3(x+7)$
14. $4x-1;\ 4(x-1)$
15. $7x-3;\ 7(x-3)$
16. $5(x+6);\ 5x+6$

Write the algebraic expressions described by the English phrases. Choose your own variable.

17. 6 added to a number
18. 7 more than a number
19. 4 less than a number
20. a number decreased by 13
21. the quotient of twice a number and 10
22. the difference between a number and 3, all divided by 7
23. 5 subtracted from three times a number
24. the sum of twice a number and four times the number
25. 8 minus twice a number
26. the sum of a number and 9 times the number
27. twenty decreased by 4.8 times a number
28. the difference between three times a number and five times the same number
29. 9 times the sum of a number and 2
30. 3 times the difference between a number and 8
31. 13 less than the product of 4 and the sum of a number and 1
32. 4 more than the product of 8 with the difference between a number and 6
33. eight more than the product of 3 and the sum of a number and 6
34. six less than twice the difference between a number and 7
35. four less than 3 times the difference between 7 and a number
36. nine more than twice the sum of 17 and a number

37. **a.** 6 less than a number

b. 6 less a number

38. **a.** 5 less than 3 times a number

b. 5 less 3 times a number

39. **a.** 20 less than a number

b. 20 less a number

40. **a.** 6 less than 4 times a number

b. 6 less 4 times a number

Write the algebraic expressions described by the English phrases.

41. ***Time:*** the number of hours in d days

42. ***Graphing Calculators:*** the cost of x graphing calculators if one calculator costs \$115

43. ***Gas Prices:*** the cost of x gallons of gasoline if the cost of one gallon is \$3.15

44. ***Time:*** the number of seconds in m minutes

45. ***Time:*** the number of days in y years. (Assume 365 days in a year.)

46. ***Candy:*** the cost of x pounds of candy at \$4.95 a pound

47. ***Time:*** the number of days in t weeks and 3 days

48. ***Time:*** the number of minutes in h hours and 20 minutes

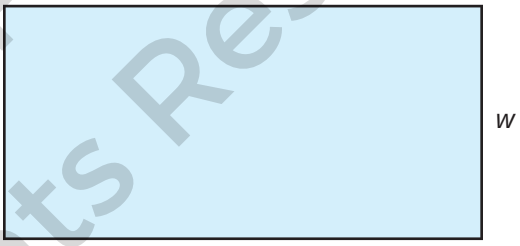
49. ***Football:*** the points scored by a football team on t touchdowns (7 points) and 1 field goal (3 points)

50. ***Vacation Time:*** the amount of vacation days an employee has after w weeks if she gets 0.2 vacation days for every week she works

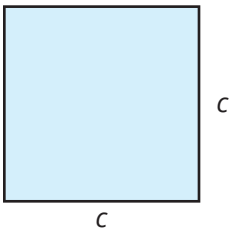
51. ***Car Rentals:*** the cost of renting a car for one day and driving m miles if the rate is \$20 per day plus 15 cents per mile

52. ***Fishing:*** the cost of purchasing a fishing rod and reel if the rod costs x dollars and the reel costs \$8 more than twice the cost of the rod

53. ***Rectangles:*** the perimeter of a rectangle if the width is w centimeters and the length is 3 cm less than twice the width



54. ***Squares:*** the area of a square with side c centimeter



Writing & Thinking

55. Discuss the meaning of the term "ambiguous phrase."