Table of Integrals

With the exception of exponent restrictions, antiderivatives in this table appear without conditions—some are valid only when appropriate restrictions on parameters (e.g., $a > 0$) and/or variables (e.g., $|x| > 1$) are applied.

Elementary Integrals

1. $\int k\,dx = kx + C$

2. $\int x^r\,dx = \dfrac{x^{r+1}}{r+1} + C, \quad r \neq -1$

3. $\int \dfrac{dx}{x} = \ln|x| + C$

4. $\int e^x\,dx = e^x + C$

5. $\int a^x\,dx = \left(\dfrac{1}{\ln a}\right)a^x + C$

6. $\int \sin x\,dx = -\cos x + C$

7. $\int \cos x\,dx = \sin x + C$

8. $\int \sec^2 x\,dx = \tan x + C$

9. $\int \csc^2 x\,dx = -\cot x + C$

10. $\int \sec x \tan x\,dx = \sec x + C$

11. $\int \csc x \cot x\,dx = -\csc x + C$

12. $\int \tan x\,dx = \ln|\sec x| + C$

13. $\int \cot x\,dx = \ln|\sin x| + C$

14. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

15. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$

Integrals with Expressions of the Form *a* + *bx*

16. $\int \dfrac{dx}{a+bx} = \dfrac{1}{b}\ln|a+bx| + C$

17. $\int \dfrac{x}{a+bx}\,dx = \dfrac{1}{b^2}\left(bx - a\ln|a+bx|\right) + C$

18. $\int \dfrac{x^2}{a+bx}\,dx = \dfrac{1}{2b^3}\left(b^2x^2 - 2abx + 2a^2\ln|a+bx|\right) + C$

19. $\int \dfrac{dx}{x(a+bx)} = \dfrac{1}{a}\ln\left|\dfrac{x}{a+bx}\right| + C$

20. $\int \dfrac{dx}{x^2(a+bx)} = -\dfrac{1}{ax} - \dfrac{b}{a^2}\ln\left|\dfrac{x}{a+bx}\right| + C$

21. $\int \dfrac{x}{(a+bx)^2}\,dx = \dfrac{1}{b^2}\left(\dfrac{a}{a+bx} + \ln|a+bx|\right) + C$

22. $\int \dfrac{dx}{x(a+bx)^2} = \dfrac{1}{a(a+bx)} + \dfrac{1}{a^2}\ln\left|\dfrac{x}{a+bx}\right| + C$

23. $\int \dfrac{x^2}{(a+bx)^2}\,dx = \dfrac{1}{b^3}\left(bx - \dfrac{a^2}{a+bx} - 2a\ln|a+bx|\right) + C$

24. $\int x\sqrt{a+bx}\,dx = \dfrac{2}{15b^2}(3bx - 2a)(a+bx)^{3/2} + C$

25. $\int \dfrac{x}{\sqrt{a+bx}}\,dx = \dfrac{2}{3b^2}(bx - 2a)\sqrt{a+bx} + C$

26. $\int \dfrac{x^2}{\sqrt{a+bx}}\,dx = \dfrac{2}{15b^3}\left(8a^2 + 3b^2x^2 - 4abx\right)\sqrt{a+bx} + C$

27. $\int \dfrac{dx}{x\sqrt{a+bx}} = \begin{cases} -\dfrac{2}{\sqrt{a}}\tanh^{-1}\sqrt{\dfrac{a+bx}{a}} + C = \dfrac{1}{\sqrt{a}}\ln\left|\dfrac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right| + C & \text{if } a > 0 \\ \dfrac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\dfrac{a+bx}{-a}} + C & \text{if } a < 0 \end{cases}$

28. $\displaystyle\int \frac{\sqrt{a+bx}}{x}\,dx = 2\sqrt{a+bx} + a\int \frac{dx}{x\sqrt{a+bx}}$

29. $\displaystyle\int \frac{\sqrt{a+bx}}{x^2}\,dx = -\frac{\sqrt{a+bx}}{x} + \frac{b}{2}\int \frac{dx}{x\sqrt{a+bx}}$

30. $\displaystyle\int x^n\sqrt{a+bx}\,dx = \frac{2}{b(2n+3)}\left[x^n(a+bx)^{3/2} - na\int x^{n-1}\sqrt{a+bx}\,dx\right]$

31. $\displaystyle\int \frac{x^n}{\sqrt{a+bx}}\,dx = \frac{2x^n\sqrt{a+bx}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{x^{n-1}}{\sqrt{a+bx}}\,dx$

32. $\displaystyle\int \frac{dx}{x^n\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{a(n-1)x^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{dx}{x^{n-1}\sqrt{a+bx}}$

Integrals with Expressions of the Form $a^2 + x^2$

33. $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

34. $\displaystyle\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

35. $\displaystyle\int x^2\sqrt{a^2+x^2}\,dx = \frac{x}{8}\left(a^2+2x^2\right)\sqrt{a^2+x^2} - \frac{a^4}{8}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

36. $\displaystyle\int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$

37. $\displaystyle\int \frac{\sqrt{a^2+x^2}}{x^2}\,dx = -\frac{\sqrt{a^2+x^2}}{x} + \ln\left(x+\sqrt{a^2+x^2}\right) + C$

38. $\displaystyle\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x+\sqrt{a^2+x^2}\right) + C = \sinh^{-1}\left(\frac{x}{a}\right) + C$

39. $\displaystyle\int \frac{x^2}{\sqrt{a^2+x^2}}\,dx = \frac{x}{2}\sqrt{a^2+x^2} - \frac{a^2}{2}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

40. $\displaystyle\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+x^2}+a}{x}\right| + C = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{x}{a}\right| + C$

41. $\displaystyle\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + C$

42. $\displaystyle\int \left(a^2+x^2\right)^{3/2}\,dx = \frac{x}{8}\left(2x^2+5a^2\right)\sqrt{a^2+x^2} + \frac{3a^4}{8}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

43. $\displaystyle\int \frac{dx}{\left(a^2+x^2\right)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} + C$

Integrals with Expressions of the Form $a^2 - x^2$

44. $$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x + a}{x - a} \right| + C = \begin{cases} \frac{1}{a} \tanh^{-1} \left(\frac{x}{a} \right) + C & \text{if } x^2 < a^2 \\ \frac{1}{a} \coth^{-1} \left(\frac{x}{a} \right) + C & \text{if } x^2 > a^2 \end{cases}$$

45. $$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

46. $$\int x^2 \sqrt{a^2 - x^2}\, dx = \frac{x}{8} \left(2x^2 - a^2 \right) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \left(\frac{x}{a} \right) + C$$

47. $$\int \frac{\sqrt{a^2 - x^2}}{x}\, dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

48. $$\int \frac{\sqrt{a^2 - x^2}}{x^2}\, dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1} \left(\frac{x}{a} \right) + C$$

49. $$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left(\frac{x}{a} \right) + C$$

50. $$\int \frac{x^2}{\sqrt{a^2 - x^2}}\, dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

51. $$\int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C = -\frac{1}{a} \operatorname{sech}^{-1} \left(\frac{x}{a} \right) + C$$

52. $$\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{1}{a^2 x} \sqrt{a^2 - x^2} + C$$

53. $$\int \left(a^2 - x^2 \right)^{3/2} dx = -\frac{x}{8} \left(2x^2 - 5a^2 \right) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \left(\frac{x}{a} \right) + C$$

54. $$\int \frac{dx}{\left(a^2 - x^2 \right)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C$$

Integrals with Expressions of the Form $x^2 - a^2$

55. $$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C$$

56. $$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right| + C$$

57. $$\int x^2 \sqrt{x^2 - a^2}\, dx = \frac{x}{8} \left(2x^2 - a^2 \right) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln \left| x + \sqrt{x^2 - a^2} \right| + C$$

58. $$\int \frac{\sqrt{x^2 - a^2}}{x}\, dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| + C$$

59. $\displaystyle\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$

60. $\displaystyle\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C = \cosh^{-1}\left(\frac{x}{a}\right) + C$

61. $\displaystyle\int \frac{x^2}{\sqrt{x^2-a^2}}\,dx = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

62. $\displaystyle\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$

63. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2 x} + C$

64. $\displaystyle\int \left(x^2-a^2\right)^{3/2}\,dx = \frac{x}{8}\left(2x^2-5a^2\right)\sqrt{x^2-a^2} + \frac{3a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

65. $\displaystyle\int \frac{dx}{\left(x^2-a^2\right)^{3/2}} = -\frac{x}{a^2\sqrt{x^2-a^2}} + C$

Integrals with Trigonometric Expressions

66. $\displaystyle\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

67. $\displaystyle\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

68. $\displaystyle\int \tan^2 x\,dx = \tan x - x + C$

69. $\displaystyle\int \cot^2 x\,dx = -\cot x - x + C$

70. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$

71. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$

72. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$

73. $\displaystyle\int \cot^n x\,dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\,dx$

74. $\displaystyle\int \sec^n x\,dx = \frac{\tan x\sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$

75. $\displaystyle\int \csc^n x\,dx = -\frac{\cot x\csc^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \csc^{n-2} x\,dx$

76. $\displaystyle\int \sin(ax)\sin(bx)\,dx = \frac{\sin\left((a-b)x\right)}{2(a-b)} - \frac{\sin\left((a+b)x\right)}{2(a+b)} + C$

77. $\displaystyle\int \cos(ax)\cos(bx)\,dx = \frac{\sin\left((a-b)x\right)}{2(a-b)} + \frac{\sin\left((a+b)x\right)}{2(a+b)} + C$

78. $\displaystyle\int \sin(ax)\cos(bx)\,dx = -\frac{\cos\left((a-b)x\right)}{2(a-b)} - \frac{\cos\left((a+b)x\right)}{2(a+b)} + C$

79. $\displaystyle\int x\sin x\,dx = \sin x - x\cos x + C$

80. $\displaystyle\int x\cos x\,dx = \cos x + x\sin x + C$

81. $\displaystyle\int x^n \sin x\,dx = -x^n\cos x + n\int x^{n-1}\cos x\,dx$

82. $\displaystyle\int x^n \cos x\,dx = x^n\sin x - n\int x^{n-1}\sin x\,dx$

83. $$\int \sin^m x \cos^n x\,dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{n+m} + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x\,dx$$
$$= \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x\,dx$$

Integrals with Inverse Trigonometric Expressions

84. $$\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1-x^2} + C$$

85. $$\int \cos^{-1} x\,dx = x\cos^{-1} x - \sqrt{1-x^2} + C$$

86. $$\int \tan^{-1} x\,dx = x\tan^{-1} x - \frac{1}{2}\ln\left(1+x^2\right) + C$$

87. $$\int \cot^{-1} x\,dx = x\cot^{-1} x + \frac{1}{2}\ln\left(x^2+1\right) + C$$

88. $$\int \sec^{-1} x\,dx = x\sec^{-1} x - \ln\left|x+\sqrt{x^2-1}\right| + C$$

89. $$\int \csc^{-1} x\,dx = x\csc^{-1} x + \ln\left|x+\sqrt{x^2-1}\right| + C$$

90. $$\int x^n \sin^{-1} x\,dx = \frac{1}{n+1}\left(x^{n+1}\sin^{-1} x - \int \frac{x^{n+1}}{\sqrt{1-x^2}}\,dx\right), \quad n \neq -1$$

91. $$\int x^n \cos^{-1} x\,dx = \frac{1}{n+1}\left(x^{n+1}\cos^{-1} x + \int \frac{x^{n+1}}{\sqrt{1-x^2}}\,dx\right), \quad n \neq -1$$

92. $$\int x^n \tan^{-1} x\,dx = \frac{1}{n+1}\left(x^{n+1}\tan^{-1} x - \int \frac{x^{n+1}}{1+x^2}\,dx\right), \quad n \neq -1$$

Integrals with Exponential or Logarithmic Expressions

93. $$\int xe^{ax}\,dx = \frac{1}{a^2}(ax-1)e^{ax} + C$$

94. $$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$

95. $$\int \sin(ax)e^{bx}\,dx = \frac{b\sin(ax) - a\cos(ax)}{a^2+b^2}e^{bx} + C$$

96. $$\int \cos(ax)e^{bx}\,dx = \frac{b\cos(ax) + a\sin(ax)}{a^2+b^2}e^{bx} + C$$

97. $$\int \ln x\,dx = x\ln x - x + C$$

98. $$\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1}\,dx$$

99. $$\int x^n \ln x\,dx = \frac{x^{n+1}}{(n+1)^2}\left[(n+1)\ln x - 1\right] + C$$

100. $$\int \frac{dx}{x\ln x} = \ln|\ln x| + C$$

Integrals with Expressions of the Form $\sqrt{2ax - x^2}$

101. $$\int \sqrt{2ax-x^2}\,dx = \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$$

102. $$\int x\sqrt{2ax-x^2}\,dx = \frac{2x^2 - ax - 3a^2}{6}\sqrt{2ax-x^2} + \frac{a^3}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$$

103. $$\int \frac{\sqrt{2ax-x^2}}{x}\,dx = \sqrt{2ax-x^2} + a\sin^{-1}\left(\frac{x-a}{a}\right) + C$$

104. $\displaystyle\int \frac{\sqrt{2ax-x^2}}{x^2}\,dx = -\frac{2}{x}\sqrt{2ax-x^2} - \sin^{-1}\left(\frac{x-a}{a}\right) + C$

105. $\displaystyle\int \frac{dx}{\sqrt{2ax-x^2}} = \sin^{-1}\left(\frac{x-a}{a}\right) + C$

106. $\displaystyle\int \frac{x}{\sqrt{2ax-x^2}}\,dx = -\sqrt{2ax-x^2} + a\sin^{-1}\left(\frac{x-a}{a}\right) + C$

107. $\displaystyle\int \frac{x^2}{\sqrt{2ax-x^2}}\,dx = -\frac{x+3a}{2}\sqrt{2ax-x^2} + \frac{3a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

108. $\displaystyle\int \frac{dx}{x\sqrt{2ax-x^2}} = -\frac{1}{ax}\sqrt{2ax-x^2} + C$

Integrals with Hyperbolic Expressions

109. $\displaystyle\int \sinh x\,dx = \cosh x + C$

110. $\displaystyle\int \cosh x\,dx = \sinh x + C$

111. $\displaystyle\int \tanh x\,dx = \ln\cosh x + C$

112. $\displaystyle\int \coth x\,dx = \ln|\sinh x| + C$

113. $\displaystyle\int \operatorname{sech} x\,dx = \tan^{-1}|\sinh x| + C$

114. $\displaystyle\int \operatorname{csch} x\,dx = \ln\left|\tanh\frac{x}{2}\right| + C$

115. $\displaystyle\int \operatorname{sech}^2 x\,dx = \tanh x + C$

116. $\displaystyle\int \operatorname{csch}^2 x\,dx = -\coth x + C$

117. $\displaystyle\int \operatorname{sech} x\tanh x\,dx = -\operatorname{sech} x + C$

118. $\displaystyle\int \operatorname{csch} x\coth x\,dx = -\operatorname{csch} x + C$

Integrals with Inverse Hyperbolic Expressions

119. $\displaystyle\int \sinh^{-1} x\,dx = x\sinh^{-1} x - \sqrt{1+x^2} + C$

120. $\displaystyle\int \cosh^{-1} x\,dx = x\cosh^{-1} x - \sqrt{x^2-1} + C$

121. $\displaystyle\int \tanh^{-1} x\,dx = x\tanh^{-1} x + \frac{1}{2}\ln\left(1-x^2\right) + C$

122. $\displaystyle\int \coth^{-1} x\,dx = x\coth^{-1} x + \frac{1}{2}\ln\left(1-x^2\right) + C$

123. $\displaystyle\int \operatorname{sech}^{-1} x\,dx = x\operatorname{sech}^{-1} x + 2\sin^{-1}\sqrt{\frac{1+x}{2}} + C$

124. $\displaystyle\int \operatorname{csch}^{-1} x\,dx = \begin{cases} x\operatorname{csch}^{-1} x + \sinh^{-1} x + C & \text{if } x > 0 \\ x\operatorname{csch}^{-1} x - \sinh^{-1} x + C & \text{if } x < 0 \end{cases}$

Selected Definite Integrals

125. $\displaystyle\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

126. $\displaystyle\int_0^\infty \frac{x}{e^x-1}\,dx = \frac{\pi^2}{6}$

127. $\displaystyle\int_0^\infty e^{-ax}\cos(bx)\,dx = \frac{a}{a^2+b^2}$

128. $\displaystyle\int_0^\infty e^{-ax}\sin(bx)\,dx = \frac{b}{a^2+b^2}$