

Table of Integrals

With the exception of exponent restrictions, antiderivatives in this table appear without conditions—some are valid only when appropriate restrictions on parameters (e.g., $a > 0$) and/or variables (e.g., $|x| > 1$) are applied.

Elementary Integrals

1. $\int k \, dx = kx + C$

3. $\int \frac{dx}{x} = \ln|x| + C$

5. $\int a^x \, dx = \left(\frac{1}{\ln a}\right)a^x + C$

7. $\int \cos x \, dx = \sin x + C$

9. $\int \csc^2 x \, dx = -\cot x + C$

11. $\int \csc x \cot x \, dx = -\csc x + C$

13. $\int \cot x \, dx = \ln|\sin x| + C$

15. $\int \csc x \, dx = \ln|\csc x - \cot x| + C$

2. $\int x^r \, dx = \frac{x^{r+1}}{r+1} + C, \quad r \neq -1$

4. $\int e^x \, dx = e^x + C$

6. $\int \sin x \, dx = -\cos x + C$

8. $\int \sec^2 x \, dx = \tan x + C$

10. $\int \sec x \tan x \, dx = \sec x + C$

12. $\int \tan x \, dx = \ln|\sec x| + C$

14. $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

Integrals with Expressions of the Form $a + bx$

16. $\int \frac{dx}{a+bx} = \frac{1}{b} \ln|a+bx| + C$

18. $\int \frac{x^2}{a+bx} \, dx = \frac{1}{2b^3} (b^2 x^2 - 2abx + 2a^2 \ln|a+bx|) + C$

19. $\int \frac{dx}{x(a+bx)} = \frac{1}{a} \ln \left| \frac{x}{a+bx} \right| + C$

21. $\int \frac{x}{(a+bx)^2} \, dx = \frac{1}{b^2} \left(\frac{a}{a+bx} + \ln|a+bx| \right) + C$

23. $\int \frac{x^2}{(a+bx)^2} \, dx = \frac{1}{b^3} \left(bx - \frac{a^2}{a+bx} - 2a \ln|a+bx| \right) + C$

24. $\int x\sqrt{a+bx} \, dx = \frac{2}{15b^2} (3bx - 2a)(a+bx)^{3/2} + C$

26. $\int \frac{x^2}{\sqrt{a+bx}} \, dx = \frac{2}{15b^3} (8a^2 + 3b^2 x^2 - 4abx) \sqrt{a+bx} + C$

$$27. \int \frac{dx}{x\sqrt{a+bx}} = \begin{cases} -\frac{2}{\sqrt{a}} \tanh^{-1} \sqrt{\frac{a+bx}{a}} + C = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right| + C & \text{if } a > 0 \\ \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bx}{-a}} + C & \text{if } a < 0 \end{cases}$$

17. $\int \frac{x}{a+bx} \, dx = \frac{1}{b^2} (bx - a \ln|a+bx|) + C$

20. $\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} - \frac{b}{a^2} \ln \left| \frac{x}{a+bx} \right| + C$

22. $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} + \frac{1}{a^2} \ln \left| \frac{x}{a+bx} \right| + C$

25. $\int \frac{x}{\sqrt{a+bx}} \, dx = \frac{2}{3b^2} (bx - 2a) \sqrt{a+bx} + C$

28. $\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} + a \int \frac{dx}{x\sqrt{a+bx}}$
29. $\int \frac{\sqrt{a+bx}}{x^2} dx = -\frac{\sqrt{a+bx}}{x} + \frac{b}{2} \int \frac{dx}{x\sqrt{a+bx}}$
30. $\int x^n \sqrt{a+bx} dx = \frac{2}{b(2n+3)} \left[x^n (a+bx)^{3/2} - na \int x^{n-1} \sqrt{a+bx} dx \right]$
31. $\int \frac{x^n}{\sqrt{a+bx}} dx = \frac{2x^n \sqrt{a+bx}}{b(2n+1)} - \frac{2na}{b(2n+1)} \int \frac{x^{n-1}}{\sqrt{a+bx}} dx$
32. $\int \frac{dx}{x^n \sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{a(n-1)x^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{dx}{x^{n-1} \sqrt{a+bx}}$

Integrals with Expressions of the Form $a^2 + x^2$

33. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$
34. $\int \sqrt{a^2+x^2} dx = \frac{x}{2} \sqrt{a^2+x^2} + \frac{a^2}{2} \ln\left(x + \sqrt{a^2+x^2}\right) + C$
35. $\int x^2 \sqrt{a^2+x^2} dx = \frac{x}{8} (a^2+2x^2) \sqrt{a^2+x^2} - \frac{a^4}{8} \ln\left(x + \sqrt{a^2+x^2}\right) + C$
36. $\int \frac{\sqrt{a^2+x^2}}{x} dx = \sqrt{a^2+x^2} - a \ln \left| \frac{a + \sqrt{a^2+x^2}}{x} \right| + C$
37. $\int \frac{\sqrt{a^2+x^2}}{x^2} dx = -\frac{\sqrt{a^2+x^2}}{x} + \ln\left(x + \sqrt{a^2+x^2}\right) + C$
38. $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x + \sqrt{a^2+x^2}\right) + C = \sinh^{-1}\left(\frac{x}{a}\right) + C$
39. $\int \frac{x^2}{\sqrt{a^2+x^2}} dx = \frac{x}{2} \sqrt{a^2+x^2} - \frac{a^2}{2} \ln\left(x + \sqrt{a^2+x^2}\right) + C$
40. $\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2+x^2} + a}{x} \right| + C = -\frac{1}{a} \operatorname{csch}^{-1} \left| \frac{x}{a} \right| + C$
41. $\int \frac{dx}{x^2 \sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2 x} + C$
42. $\int (a^2+x^2)^{3/2} dx = \frac{x}{8} (2x^2+5a^2) \sqrt{a^2+x^2} + \frac{3a^4}{8} \ln\left(x + \sqrt{a^2+x^2}\right) + C$
43. $\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2+x^2}} + C$

Integrals with Expressions of the Form $a^2 - x^2$

$$44. \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C = \begin{cases} \frac{1}{a} \tanh^{-1} \left(\frac{x}{a} \right) + C & \text{if } x^2 < a^2 \\ \frac{1}{a} \coth^{-1} \left(\frac{x}{a} \right) + C & \text{if } x^2 > a^2 \end{cases}$$

$$45. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$46. \int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$47. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$48. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$49. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$50. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$51. \int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C = -\frac{1}{a} \operatorname{sech}^{-1} \left(\frac{x}{a} \right) + C$$

$$52. \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{1}{a^2 x} \sqrt{a^2 - x^2} + C$$

$$53. \int (a^2 - x^2)^{3/2} dx = -\frac{x}{8} (2x^2 - 5a^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$54. \int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C$$

Integrals with Expressions of the Form $x^2 - a^2$

$$55. \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$$

$$56. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$57. \int x^2 \sqrt{x^2 - a^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$58. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| + C$$

59. $\int \frac{\sqrt{x^2 - a^2}}{x^2} dx = -\frac{\sqrt{x^2 - a^2}}{x} + \ln|x + \sqrt{x^2 - a^2}| + C$
60. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C = \cosh^{-1}\left(\frac{x}{a}\right) + C$
61. $\int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{x}{2}\sqrt{x^2 - a^2} + \frac{a^2}{2}\ln|x + \sqrt{x^2 - a^2}| + C$
62. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
63. $\int \frac{dx}{x^2\sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$
64. $\int (x^2 - a^2)^{3/2} dx = \frac{x}{8}(2x^2 - 5a^2)\sqrt{x^2 - a^2} + \frac{3a^4}{8}\ln|x + \sqrt{x^2 - a^2}| + C$
65. $\int \frac{dx}{(x^2 - a^2)^{3/2}} = -\frac{x}{a^2\sqrt{x^2 - a^2}} + C$

Integrals with Trigonometric Expressions

66. $\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$
67. $\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$
68. $\int \tan^2 x dx = \tan x - x + C$
69. $\int \cot^2 x dx = -\cot x - x + C$
70. $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
71. $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
72. $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$
73. $\int \cot^n x dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx$
74. $\int \sec^n x dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$
75. $\int \csc^n x dx = -\frac{\cot x \csc^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x dx$
76. $\int \sin(ax)\sin(bx) dx = \frac{\sin((a-b)x)}{2(a-b)} - \frac{\sin((a+b)x)}{2(a+b)} + C$
77. $\int \cos(ax)\cos(bx) dx = \frac{\sin((a-b)x)}{2(a-b)} + \frac{\sin((a+b)x)}{2(a+b)} + C$
78. $\int \sin(ax)\cos(bx) dx = -\frac{\cos((a-b)x)}{2(a-b)} - \frac{\cos((a+b)x)}{2(a+b)} + C$
79. $\int x \sin x dx = \sin x - x \cos x + C$
80. $\int x \cos x dx = \cos x + x \sin x + C$
81. $\int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$
82. $\int x^n \cos x dx = x^n \sin x - n \int x^{n-1} \sin x dx$

$$\begin{aligned}
 83. \int \sin^m x \cos^n x \, dx &= -\frac{\sin^{m-1} x \cos^{n+1} x}{n+m} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x \, dx \\
 &= \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x \, dx
 \end{aligned}$$

Integrals with Inverse Trigonometric Expressions

$$\begin{aligned}
 84. \int \sin^{-1} x \, dx &= x \sin^{-1} x + \sqrt{1-x^2} + C & 85. \int \cos^{-1} x \, dx &= x \cos^{-1} x - \sqrt{1-x^2} + C \\
 86. \int \tan^{-1} x \, dx &= x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C & 87. \int \cot^{-1} x \, dx &= x \cot^{-1} x + \frac{1}{2} \ln(x^2+1) + C \\
 88. \int \sec^{-1} x \, dx &= x \sec^{-1} x - \ln|x + \sqrt{x^2-1}| + C & 89. \int \csc^{-1} x \, dx &= x \csc^{-1} x + \ln|x + \sqrt{x^2-1}| + C \\
 90. \int x^n \sin^{-1} x \, dx &= \frac{1}{n+1} \left(x^{n+1} \sin^{-1} x - \int \frac{x^{n+1}}{\sqrt{1-x^2}} \, dx \right), \quad n \neq -1 \\
 91. \int x^n \cos^{-1} x \, dx &= \frac{1}{n+1} \left(x^{n+1} \cos^{-1} x + \int \frac{x^{n+1}}{\sqrt{1-x^2}} \, dx \right), \quad n \neq -1 \\
 92. \int x^n \tan^{-1} x \, dx &= \frac{1}{n+1} \left(x^{n+1} \tan^{-1} x - \int \frac{x^{n+1}}{1+x^2} \, dx \right), \quad n \neq -1
 \end{aligned}$$

Integrals with Exponential or Logarithmic Expressions

$$\begin{aligned}
 93. \int x e^{ax} \, dx &= \frac{1}{a^2} (ax-1) e^{ax} + C & 94. \int x^n e^{ax} \, dx &= \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx \\
 95. \int \sin(ax) e^{bx} \, dx &= \frac{b \sin(ax) - a \cos(ax)}{a^2 + b^2} e^{bx} + C & 96. \int \cos(ax) e^{bx} \, dx &= \frac{b \cos(ax) + a \sin(ax)}{a^2 + b^2} e^{bx} + C \\
 97. \int \ln x \, dx &= x \ln x - x + C & 98. \int (\ln x)^n \, dx &= x (\ln x)^n - n \int (\ln x)^{n-1} \, dx \\
 99. \int x^n \ln x \, dx &= \frac{x^{n+1}}{(n+1)^2} [(n+1) \ln x - 1] + C & 100. \int \frac{dx}{x \ln x} &= \ln|\ln x| + C
 \end{aligned}$$

Integrals with Expressions of the Form $\sqrt{2ax-x^2}$

$$\begin{aligned}
 101. \int \sqrt{2ax-x^2} \, dx &= \frac{x-a}{2} \sqrt{2ax-x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + C \\
 102. \int x \sqrt{2ax-x^2} \, dx &= \frac{2x^2-ax-3a^2}{6} \sqrt{2ax-x^2} + \frac{a^3}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + C \\
 103. \int \frac{\sqrt{2ax-x^2}}{x} \, dx &= \sqrt{2ax-x^2} + a \sin^{-1} \left(\frac{x-a}{a} \right) + C
 \end{aligned}$$

$$104. \int \frac{\sqrt{2ax-x^2}}{x^2} dx = -\frac{2}{x}\sqrt{2ax-x^2} - \sin^{-1}\left(\frac{x-a}{a}\right) + C$$

$$105. \int \frac{dx}{\sqrt{2ax-x^2}} = \sin^{-1}\left(\frac{x-a}{a}\right) + C$$

$$106. \int \frac{x}{\sqrt{2ax-x^2}} dx = -\sqrt{2ax-x^2} + a \sin^{-1}\left(\frac{x-a}{a}\right) + C$$

$$107. \int \frac{x^2}{\sqrt{2ax-x^2}} dx = -\frac{x+3a}{2}\sqrt{2ax-x^2} + \frac{3a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$$

$$108. \int \frac{dx}{x\sqrt{2ax-x^2}} = -\frac{1}{ax}\sqrt{2ax-x^2} + C$$

Integrals with Hyperbolic Expressions

$$109. \int \sinh x dx = \cosh x + C$$

$$110. \int \cosh x dx = \sinh x + C$$

$$111. \int \tanh x dx = \ln \cosh x + C$$

$$112. \int \coth x dx = \ln |\sinh x| + C$$

$$113. \int \operatorname{sech} x dx = \tan^{-1} |\sinh x| + C$$

$$114. \int \operatorname{csch} x dx = \ln \left| \tanh \frac{x}{2} \right| + C$$

$$115. \int \operatorname{sech}^2 x dx = \tanh x + C$$

$$116. \int \operatorname{csch}^2 x dx = -\coth x + C$$

$$117. \int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$$

$$118. \int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + C$$

Integrals with Inverse Hyperbolic Expressions

$$119. \int \sinh^{-1} x dx = x \sinh^{-1} x - \sqrt{1+x^2} + C$$

$$120. \int \cosh^{-1} x dx = x \cosh^{-1} x - \sqrt{x^2-1} + C$$

$$121. \int \tanh^{-1} x dx = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) + C$$

$$122. \int \coth^{-1} x dx = x \coth^{-1} x + \frac{1}{2} \ln(1-x^2) + C$$

$$123. \int \operatorname{sech}^{-1} x dx = x \operatorname{sech}^{-1} x + 2 \sin^{-1} \sqrt{\frac{1+x}{2}} + C$$

$$124. \int \operatorname{csch}^{-1} x dx = \begin{cases} x \operatorname{csch}^{-1} x + \sinh^{-1} x + C & \text{if } x > 0 \\ x \operatorname{csch}^{-1} x - \sinh^{-1} x + C & \text{if } x < 0 \end{cases}$$

Selected Definite Integrals

$$125. \int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

$$126. \int_0^{\infty} \frac{x}{e^x - 1} dx = \frac{\pi^2}{6}$$

$$127. \int_0^{\infty} e^{-ax} \cos(bx) dx = \frac{a}{a^2 + b^2}$$

$$128. \int_0^{\infty} e^{-ax} \sin(bx) dx = \frac{b}{a^2 + b^2}$$