Formulas and Tables from *Beginning Statistics*

Chapter 2

Class Midpoint

$$\frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

Relative Frequency

$$\frac{\text{Class Frequency}}{\text{Sample Size}} = \frac{f}{n}$$

Chapter 3

Sample Mean

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum x_i}{n}$$

Population Mean

$$\mu = \frac{x_1 + x_2 + \cdots + x_N}{N} = \frac{\sum x_i}{N}$$

Weighted Mean

$$\bar{x} = \frac{\sum (x_i \cdot w_i)}{\sum w_i}$$

Range

Maximum Data Value – Minimum Data Value

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

Sample Standard Deviation

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

Population Coefficient of Variation

$$\text{CV} = \frac{\sigma}{\mu} \cdot 100\%$$

Sample Coefficient of Variation

$$\text{CV} = \frac{s}{\bar{x}} \cdot 100\%$$

Population Variance

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

Sample Variance

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Empirical Rule for Bell-Shaped Distributions

Approximately 68% of the data values lie within one standard deviation of the mean.

Approximately 95% of the data values lie within two standard deviations of the mean.

Approximately 99.7% of the data values lie within three standard deviations of the mean.

Chebyshev's Theorem

The proportion of any data set lying within K standard deviations of the mean is at least $1 - \frac{1}{K^2}$ for $K > 1$.

Location of Data Value for the P^{th} Percentile

$$l = n \cdot \frac{P}{100}$$

P^{th} Percentile of a Data Value

$$P = \frac{l}{n} \cdot 100$$

Quartiles

Q_1 = First Quartile: 25% of the data are less than or equal to this value.

Q_2 = Second Quartile: 50% of the data are less than or equal to this value.

Q_3 = Third Quartile: 75% of the data are less than or equal to this value.

Interquartile Range

$$\text{IQR} = Q_3 - Q_1$$

Standard Score for a Population

$$z = \frac{x - \mu}{\sigma}$$

Standard Score for a Sample

$$z = \frac{x - \bar{x}}{s}$$

Chapter 4

Experimental Probability (or Empirical Probability)

$$P(E)=\frac{f}{n}$$

Classical Probability (or Theoretical Probability)

$$P(E)=\frac{n(E)}{n(S)}$$

Complement Rule for Probability

$$P(E)+P(E^c)=1$$

Addition Rule for Probability

$$P(E \textbf{ or } F)=P(E)+P(F)-P(E \text{ and } F)$$

Addition Rule for Probability of Mutually Exclusive Events

$$P(E \textbf{ or } F)=P(E)+P(F)$$

Multiplication Rule for Probability of Independent Events

$$P(E \textbf{ and } F)=P(E)\cdot P(F)$$

Multiplication Rule for Probability of Dependent Events

$$\begin{aligned} P(E \textbf{ and } F) &= P(E)\cdot P(F \mid E) \\ &= P(F)\cdot P(E \mid F) \end{aligned}$$

Conditional Probability

$$P(F \mid E)=\frac{P(E \textbf{ and } F)}{P(E)}$$

Fundamental Counting Principle

The total number of possible outcomes for the sequence of stages in a multistage experiment is $k_1 \cdot k_2 \cdot \cdots \cdot k_n$.

Factorial

$$n!=n(n-1)(n-2)\cdots(2)(1)$$

Combinations

$${}_nC_r=\frac{n!}{r!(n-r)!}$$

Permutations

$${}_nP_r=\frac{n!}{(n-r)!}$$

Special Permutations

$$\frac{n!}{k_1!k_2!\cdots k_p!}$$

Chapter 5

Expected Value

$$E(X)=\mu=\sum\left[x_i \cdot P(X=x_i)\right]$$

Variance for a Discrete Probability Distribution

$$\begin{aligned} \sigma^2 &= \sum\left[x_i^2 \cdot P(X=x_i)\right]-\mu^2 \\ &= \sum\left[(x_i-\mu)^2 \cdot P(X=x_i)\right] \end{aligned}$$

Standard Deviation for a Discrete Probability Distribution

$$\begin{aligned} \sigma &= \sqrt{\sigma^2} \\ &= \sqrt{\sum\left[x_i^2 \cdot P(X=x_i)\right]-\mu^2} \\ &= \sqrt{\sum\left[(x_i-\mu)^2 \cdot P(X=x_i)\right]} \end{aligned}$$

Probability for a Binomial Distribution

$$P(X=x)={}_nC_x \cdot p^x(1-p)^{(n-x)}$$

Probability for a Poisson Distribution

$$P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$$

Probability for a Hypergeometric Distribution

$$P(X=x)=\frac{({}_kC_x)({}_{N-k}C_{n-x})}{({}_NC_n)}$$

Chapter 6

Standard Score

$$z=\frac{x-\mu}{\sigma}$$

Finding the Value of a Normally Distributed Random Variable for a Given Probability

$$x=z\cdot\sigma+\mu$$

Normal Distribution Approximation of a Binomial Distribution

$$\mu=np$$

$$\sigma=\sqrt{np(1-p)}$$

Chapter 7

The Central Limit Theorem (CLT)

1. Mean of a Sampling Distribution of Sample Means

$$\mu_{\bar{x}} = \mu$$

2. Standard Deviation of a Sampling Distribution of Sample Means

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

3. The shape of a sampling distribution of sample means will approach that of a normal distribution, regardless of the shape of the population distribution. The larger the sample size, the better the normal distribution approximation will be.

Standard Score for a Sample Mean

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

Population Proportion

$$p = \frac{x}{N}$$

Sample Proportion

$$\hat{p} = \frac{x}{n}$$

Mean of a Sampling Distribution of Sample Proportions

$$\mu_{\hat{p}} = p$$

Standard Deviation of a Sampling Distribution of Sample Proportions

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Standard Score for a Sample Proportion

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Chapter 8

Margin of Error of a Confidence Interval for a Population Mean (σ Known)

$$E = \left(z_{\alpha/2}\right)\left(\sigma_{\bar{x}}\right) = \left(z_{\alpha/2}\right)\left(\frac{\sigma}{\sqrt{n}}\right)$$

Confidence Interval for a Population Mean

$$\bar{x} - E < \mu < \bar{x} + E$$

or

$$\left(\bar{x} - E,\ \bar{x} + E\right)$$

Minimum Sample Size for Estimating a Population Mean

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

Margin of Error of a Confidence Interval for a Population Mean (σ Unknown)

$$E = \left(t_{\alpha/2}\right)\left(\frac{s}{\sqrt{n}}\right) \text{ with } df = n - 1$$

Margin of Error of a Confidence Interval for a Population Proportion

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$$

Confidence Interval for a Population Proportion

$$\hat{p} - E < p < \hat{p} + E$$

or

$$\left(\hat{p} - E,\ \hat{p} + E\right)$$

Minimum Sample Size for Estimating a Population Proportion

$$n = p(1-p)\left(\frac{z_{\alpha/2}}{E}\right)^2$$

Confidence Interval for a Population Variance

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}} \text{ with } df = n - 1$$

Confidence Interval for a Population Standard Deviation

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}} \text{ with } df = n - 1$$

Chapter 9

Margin of Error of a Confidence Interval for the Difference between Two Population Means (σ Known)

$$E = z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

Confidence Interval for the Difference between Two Population Means

$$(\bar{x}_1-\bar{x}_2)-E < \mu_1-\mu_2 < (\bar{x}_1-\bar{x}_2)+E$$

or

$$\left((\bar{x}_1-\bar{x}_2)-E,\ (\bar{x}_1-\bar{x}_2)+E\right)$$

Margin of Error of a Confidence Interval for the Difference between Two Population Means (σ Unknown, Unequal Variances)

$$E = t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

with df = smaller of the values n_1-1 and n_2-1

Margin of Error of a Confidence Interval for the Difference between Two Population Means (σ Unknown, Equal Variances)

$$E = t_{\alpha/2}\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

with $df = n_1+n_2-2$

Paired Difference

$$d = x_2 - x_1$$

Mean of Paired Differences

$$\bar{d} = \frac{\sum d_i}{n}$$

Sample Standard Deviation of Paired Differences

$$s_d = \sqrt{\frac{\sum (d_i-\bar{d})^2}{n-1}}$$

Margin of Error of a Confidence Interval for the Mean of the Paired Differences for Two Populations (σ Unknown, Dependent Samples)

$$E = (t_{\alpha/2})\left(\frac{s_d}{\sqrt{n}}\right) \text{ with } df = n-1$$

Confidence Interval for the Mean of the Paired Differences for Two Populations (Dependent Samples)

$$\bar{d}-E < \mu_d < \bar{d}+E$$

or

$$(\bar{d}-E,\ \bar{d}+E)$$

Margin of Error of a Confidence Interval for the Difference between Two Population Proportions

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Confidence Interval for the Difference between Two Population Proportions

$$(\hat{p}_1-\hat{p}_2)-E < p_1-p_2 < (\hat{p}_1-\hat{p}_2)+E$$

or

$$\left((\hat{p}_1-\hat{p}_2)-E,\ (\hat{p}_1-\hat{p}_2)+E\right)$$

Point Estimate for Comparing Two Population Variances

$$\frac{s_1^2}{s_2^2} \text{ with } s_1^2 \geq s_2^2$$

Confidence Interval for the Ratio of Two Population Variances

$$\left(\frac{s_1^2}{s_2^2}\cdot\frac{1}{F_{\alpha/2}}\right) < \frac{\sigma_1^2}{\sigma_2^2} < \left(\frac{s_1^2}{s_2^2}\cdot\frac{1}{F_{(1-\alpha/2)}}\right)$$

with $df_1 = n_1-1$ and $df_2 = n_2-1$

Confidence Interval for the Ratio of Two Population Standard Deviations

$$\left(\frac{s_1}{s_2}\cdot\frac{1}{\sqrt{F_{\alpha/2}}}\right) < \frac{\sigma_1}{\sigma_2} < \left(\frac{s_1}{s_2}\cdot\frac{1}{\sqrt{F_{(1-\alpha/2)}}}\right)$$

with $df_1 = n_1-1$ and $df_2 = n_2-1$

Chapter 10

Level of Significance

$$\alpha = 1-c$$

Test Statistic for a Hypothesis Test for a Population Mean (σ Known)

$$z = \frac{\bar{x}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

Test Statistic for a Hypothesis Test for a Population Mean (σ Unknown)

$$t = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)} \text{ with } df = n - 1$$

Test Statistic for a Hypothesis Test for a Population Proportion

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Test Statistic for a Hypothesis Test for a Population Variance or Population Standard Deviation

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \text{ with } df = n - 1$$

Test Statistic for a Chi-Square Test for Goodness of Fit

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \text{ with } df = k - 1$$

Expected Value of a Frequency in a Contingency Table

$$E_i = \frac{(\text{row total})(\text{column total})}{n}$$

Test Statistic for a Chi-Square Test for Association

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \text{ with } df = (R-1)\cdot(C-1)$$

Chapter 11

Test Statistic for a Hypothesis Test For Two Population Means (σ Known)

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Test Statistic for a Hypothesis Test For Two Population Means (σ Unknown, Unequal Variances)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

with df = smaller of the values $n_1 - 1$ and $n_2 - 1$

Test Statistic for a Hypothesis Test For Two Population Means (σ Unknown, Equal Variances)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

with $df = n_1 + n_2 - 2$

Test Statistic for a Hypothesis Test for the Mean of the Paired Differences for Two Populations (σ Unknown, Dependent Samples)

$$t = \frac{\bar{d} - \mu_d}{\left(\frac{s_d}{\sqrt{n}}\right)} \text{ with } df = n - 1$$

Test Statistic for a Hypothesis Test for Two Population Proportions

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

Weighted Estimate of the Common Population Proportion

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

Test Statistic for a Hypothesis Test for Two Population Variances

$$F = \frac{s_1^2}{s_2^2} \text{ with } df_1 = n_1 - 1 \text{ and } df_2 = n_2 - 1$$

Grand Mean

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k}(n_i \bar{x}_i)}{\sum_{i=1}^{k} n_i}$$

Sum of Squares among Treatments (SST)

$$\text{SST} = \sum_{i=1}^{k} n_i (\bar{x}_i - \bar{\bar{x}})^2$$

Sum of Squares for Error (SSE)

$$\text{SSE} = \sum_{j=1}^{n_1}(x_{1j} - \bar{x}_1)^2 + \sum_{j=1}^{n_2}(x_{2j} - \bar{x}_2)^2 + \cdots + \sum_{j=1}^{n_k}(x_{kj} - \bar{x}_k)^2$$

Total Variation

$$\sum_{j=1}^{n_1}(x_{1j} - \bar{\bar{x}})^2 + \sum_{j=1}^{n_2}(x_{2j} - \bar{\bar{x}})^2 + \cdots + \sum_{j=1}^{n_k}(x_{kj} - \bar{\bar{x}})^2$$
$$= \text{SST} + \text{SSE}$$

Mean Square for Treatments (MST)

$$\text{MST} = \frac{\text{SST}}{\text{DFT}} \text{ with DFT} = k-1$$

Mean Square for Error (MSE)

$$\text{MSE} = \frac{\text{SSE}}{\text{DFE}} \text{ with DFE} = n_T - k$$

Test Statistic for an ANOVA Test

$$F = \frac{\text{MST}}{\text{MSE}} \text{ with } df_1 = \text{DFT} = k-1 \text{ and } df_2 = \text{DFE} = n_T - k$$

Chapter 12

Pearson Correlation Coefficient

$$r = \frac{n\sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)}{\sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2}\sqrt{n\sum y_i^2 - \left(\sum y_i\right)^2}}$$

$$\text{such that } -1 \le r \le 1$$

Test Statistic for a Hypothesis Test for a Correlation Coefficient

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} \text{ with } df = n-2$$

Slope of the Least-Squares Regression Line

$$b_1 = \frac{n\sum x_i y_i - \left(\sum x_i\right)\left(\sum y_i\right)}{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

y-Intercept of the Least-Squares Regression Line

$$b_0 = \frac{\sum y_i}{n} - b_1 \frac{\sum x_i}{n}$$

Regression Line (Line of Best Fit)

$$y = \beta_0 + \beta_1 x \text{ (Population parameters)}$$

$$\hat{y} = b_0 + b_1 x \text{ (Sample statistics)}$$

Residual

$$y - \hat{y}$$

Sum of Squared Errors (SSE)

$$\text{SSE} = \sum\left(y_i - \hat{y}_i\right)^2$$

Standard Error of Estimate

$$S_e = \sqrt{\frac{\sum\left(y_i - \hat{y}_i\right)^2}{n-2}} = \sqrt{\frac{\text{SSE}}{n-2}}$$

Margin of Error for a Prediction Interval for an Individual y-Value

$$E = t_{\alpha/2} S_e \sqrt{1 + \frac{1}{n} + \frac{n\left(x_0 - \bar{x}\right)^2}{n\left(\sum x_i^2\right) - \left(\sum x_i\right)^2}}$$

Prediction Interval for an Individual y-Value

$$\hat{y} - E < y < \hat{y} + E$$

or

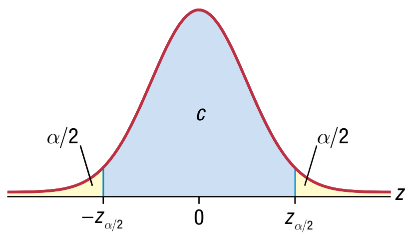
$$\left(\hat{y} - E,\ \hat{y} + E\right)$$

Multiple Regression Model

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_k x_k$$

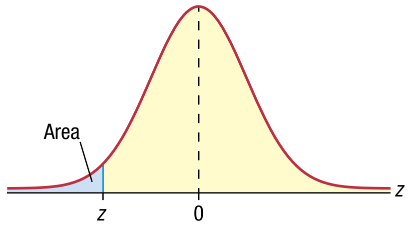
Critical Values of z

Level of Confidence, c	$\alpha = 1 - c$	$z_{\alpha/2}$
0.80	0.20	1.28
0.85	0.15	1.44
0.90	0.10	1.645
0.95	0.05	1.96
0.98	0.02	2.33
0.99	0.01	2.575



A—Standard Normal Distribution

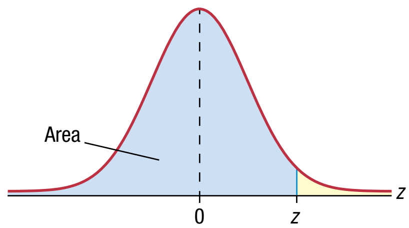
Numerical entries represent the probability that a standard normal random variable is between $-\infty$ and z where $z = \frac{x-\mu}{\sigma}$.



z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
–3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
–3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
–3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
–3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
–3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
–2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
–2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
–2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
–2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
–2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
–2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
–2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
–2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
–2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
–2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
–1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
–1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
–1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
–1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
–1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
–1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
–1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
–1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
–1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
–1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
–0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
–0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
–0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
–0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
–0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
–0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
–0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
–0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
–0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
–0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

B—Standard Normal Distribution

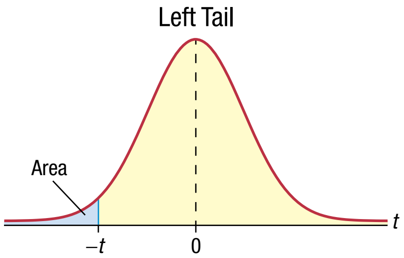
Numerical entries represent the probability that a standard normal random variable is between $-\infty$ and z where $z = \frac{x - \mu}{\sigma}$.

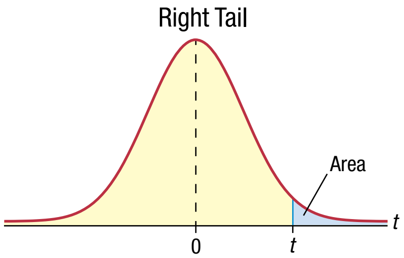


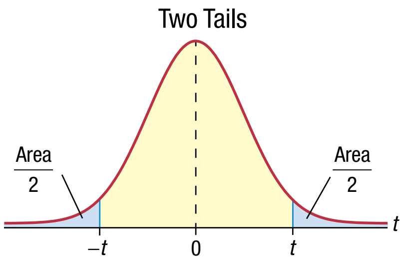
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

C—Critical Values of *t*

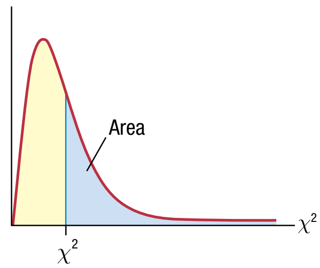
	Area in One Tail				
	0.100	0.050	0.025	0.010	0.005
	Area in Two Tails				
df	**0.200**	**0.100**	**0.050**	**0.020**	**0.010**
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
120	1.289	1.658	1.980	2.358	2.617
200	1.286	1.653	1.972	2.345	2.601
300	1.284	1.650	1.968	2.339	2.592
400	1.284	1.649	1.966	2.336	2.588
500	1.283	1.648	1.965	2.334	2.586
750	1.283	1.647	1.963	2.331	2.582
1000	1.282	1.646	1.962	2.330	2.581
∞	1.282	1.645	1.960	2.326	2.576







G—Critical Values of χ^2



Area to the Right of the Critical Value of χ^2

df	0.995	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169