

PRECALCULUS

GUIDED NOTES



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A Message to the Student

Have you ever been working on your homework and knew you had some notes somewhere for a problem you were stuck on, but didn't know where your notes were? Have you ever tried to enter an answer on a homework program by guessing and didn't know for sure why you got it wrong (or right)? Have you ever been working on math and wanted to throw your computer out the window? If you answered "yes" to any of these questions, then this notebook is for you. In my experiences as a math instructor, I have seen many students struggle with the material because of a lack of organization and motivation. This notebook was written to help students that struggle with these problems.

This notebook is designed to be used in conjunction with the Hawkes Learning *Precalculus* courseware. After a brief introduction to the concepts and objectives, each section of this workbook contains three main components: 1) **Learn**; 2) **Video Examples**; and 3) **Practice**. **Learn** will guide you through the topics that will be covered in the section and corresponds to the *Learn* mode on your courseware. **Video Examples** allows you the opportunity to solve problems that are similar to ones found on www.hawkestv.com. In **Practice**, you will work through additional examples that are similar to problems you will find in the *Practice* mode of the courseware. At this point, you will be ready to work through the additional *Practice* problems, putting yourself in a great position to succeed when attempting to *Certify* on that section in the courseware.

When used in this way, this notebook can be a valuable asset in helping you stay organized, motivated, and on track to succeed in your Precalculus course.

Chris Schroeder
Morehead State University

Important Information

Name: _____

Class Time: _____

Office Hours: _____

Teacher's phone: _____

Teacher's email: _____

Exam Dates: _____

Final Exam Date: _____

Hawkes Website: learn.hawkeslearning.com

Videos Website: tv.hawkeslearning.com

Notes:

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Schedule

Month:						
S	M	T	W	T	F	S
Notes:						

Month:						
S	M	T	W	T	F	S
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My Habits for Success in Precalculus

Students who have consistent and effective study habits are far more likely to succeed in their academic work. Research shows that writing down what these habits will be prior to using them greatly increases the chances that you will utilize these habits effectively. With that in mind, take some time to write out what habits you will develop and use to help you succeed in this course. You can do this!

My Study Habits

1. I will begin working on each lesson at least _____ days before the Certification for that lesson is due. Specifically, when working through a lesson, I will use the following habits:
2. I will begin studying for exams at least _____ days before taking the exam. Specifically, when studying for an exam, I will use the following habits:

My Habits When Things Get Rough

1. I will do my best to attend every class. However, if I have to miss a class, to catch up on the material missed I will use the following habits:
2. If I am stuck on a lesson, I will use the following habits to get help and figure out the material:

Chapter 1

Number Systems and Equations of One Variable

Before we can embark on a journey into algebra, we need to be sure that we know the players and the rules of the game. The players are the numbers, and there are many different types of numbers. The rules tell us how to work with these numbers. Most of these you probably already know, but it's worth going over them again to make sure you're ready for algebra.

1.1 Real Numbers and Algebraic Expressions

Most of what you do in this class will be done using real numbers and working with the rules of algebra that are used in the real number system. Having a good understanding of these numbers and these rules will go a long way towards ensuring your success in this class.

1.1.a The Real Number System

The real numbers contain within them many different types of numbers. Our goal in this section is to be able to classify the different types of real numbers according to their properties.

Learn:

1. Give an example of:

(a) a number which is an integer but *not* a natural number.

(b) a number which is a rational number but *not* an integer.

(c) two irrational numbers.

2. Fill in the blank with one of the symbols $<$ or $>$:

(a) -4 _____ -14

(b) 7 _____ -8

(c) $\sqrt{2}$ _____ $\sqrt{5}$

3. List three numbers which are in the set $\{x|x \text{ is an integer and } -1 \leq x < 3\}$

4. Write the set $\{x|x \geq 3\}$ in interval notation.

5. Given two real numbers a and b , the _____ between them is defined to be $|a - b|$.

Video Examples:

Watch the videos for Section 1.1a on www.hawkestv.com and then solve the following problems.

- Write the following set as an interval using interval notation: $\{x \mid -5 \leq x < 3\}$.

- Find the distance between the following numbers on the real number line.

$$a = -3, b = -10$$

Practice:

1. Identify the following number. Choose all that apply.

$$-\frac{3}{4}$$

Natural Number; Whole Number; Integer; Rational Number; Irrational Number; Real Number; Undefined

2. Evaluate the expression.

$$-|2 - 5|$$

3. Plot the set $\{-2.5, 3, 1.5, 0\}$ on the number line.

Remember to go through all of the problems in the Practice section before attempting to Certify!

1.1.b The Arithmetic of Algebraic Expressions

In this section we look at the terminology and the rules involved in working with algebraic expressions. The material in this section is fundamental to algebra.

Learn:

1. **Algebraic expressions** are made up of *constants* and _____, combined by the operations of addition, subtraction, multiplication, division, exponentiation, and the taking of _____.
2. Write the terms separately for the algebraic expression $3xy^2 - 7x^2z^3 + 4(xy - z)$.
3. Write the additive and multiplicative versions of the Commutative Property.
4. In the expression $3 - 2 \cdot 4 + 7$, which operation should be performed first: subtraction, multiplication or division?
Simplify the expression.
5. Simplify each of the following set expressions, if possible.
 - (a) $(-3, 2] \cup [1, 6]$
 - (b) $(-3, 2] \cap [1, 6]$

Video Examples:

Watch the videos for Section 1.1b on www.hawkestv.com and then solve the following problems.

- Evaluate the following expression for the given values of the variables:

$$|x + 9y| - (5z + 6) \text{ for } x = -2, y = -1, \text{ and } z = 5$$

- Evaluate the following expression.

$$\frac{8 - 3 \cdot 4 - 5}{-2(-7 - 6 \div (1 + 2))}$$

Practice:

1. Evaluate the following expression for the given values of the variables:

$$3\sqrt{x-3} + 7y^3 \text{ for } x = 28 \text{ and } y = -3$$

2. Simplify the following intersection of intervals:

$$(-\infty, -13] \cap [-13, 6)$$

3. Identify the property that justifies the following statement:

$$-4(6y + 1) = -24y - 4$$

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Remember to go through all of the problems in the Practice section before attempting to Certify!

1.2 Properties of Exponents and Radicals

Being able to evaluate expressions with exponents is an important skill to have to be successful in algebra. In the first part of this section, we'll review the properties we use to work with exponents. Then, we'll apply those properties to scientific notation and geometric problems.

1.2.a Properties of Exponents

The properties of exponents are a set of rules that allow us to evaluate and simplify exponential expressions. It's important that we have a solid working knowledge of these properties.

Learn:

1. In the expression a^n , a is called the _____, and n is the _____.

2. Complete the following list of properties of exponents.

(a) $a^n \cdot a^m =$

(b) $\frac{a^n}{a^m} =$

(c) $a^{-n} =$

(d) $(a^n)^m =$

(e) $(ab)^n =$

(f) $\left(\frac{a}{b}\right)^n =$

3. If $a \neq 0$, then $a^0 =$ _____.

Video Examples:

Watch the videos for Section 1.2a on www.hawkestv.com and then solve the following problems.

- Simplify the expression, writing your answer with only positive exponents.

$$\frac{x^4 \cdot x^7}{x^3}$$

- Simplify the expression, writing your answer with only positive exponents.

$$\left[\frac{y^6 (xy^2)^{-3}}{3x^{-3}z} \right]^{-2}$$

Practice:

1. Simplify the expression, writing your answer with only positive exponents.

$$(2y^{-1})^{-1}$$

2. Simplify the expression, writing your answer with only positive exponents.

$$\left(\frac{2a}{b^{-3}}\right)^2$$

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3. Use the properties of exponents to simplify the expression, writing your answer with only positive exponents.

$$\left((3x^{-2}y^4)^3\right)^{-1}$$

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Remember to go through all of the problems in the Practice section before attempting to Certify!

1.2.b Scientific Notation and Geometric Problems Using Exponents

Scientific notation allows scientists to more easily work with very large or very small numbers using properties of exponents. Of course, geometric problems occur in many areas and exponents are often involved in these formulas.

Learn:

1. For a number in scientific notation of the form $a \times 10^n$, between what two numbers must the absolute value of a be?

$$\underline{\hspace{1cm}} \leq a < \underline{\hspace{1cm}}$$

2. To convert a number in scientific notation to a number in decimal notation, we move the decimal in n units to the _____ if n is positive and to the _____ if n is negative.

3. Write the geometric formula for the following:

(a) Volume of a sphere: $V =$

(b) Surface area of a box: $S =$

(c) Surface area of a right circular cylinder: $S =$

(d) Volume of a right trapezoidal cylinder: $V =$

Video Examples:

Watch the videos for Section 1.2b on www.hawkestv.com and then solve the following problems.

- Evaluate the following expression, using the properties of exponents. Write your answer in standard notation.

$$\frac{(15.3 \times 10^{15})(3 \times 10^{-11})}{(5.1 \times 10^9)}$$

- Determine the volume of a right trapezoidal cylinder whose bases are $B = 18$ m and $b = 9$ m, height is $h = 4$ m, and length is $l = 33$ m.

Practice:

1. Convert the following number from scientific notation to standard notation.

$$7.32 \times 10^5$$

2. Evaluate the following expression, using the properties of exponents. Write your answer in scientific notation.

$$(4 \times 10^{-6}) (3.7 \times 10^{-7}) (7 \times 10^{-11})$$

3. Determine the surface area of a box whose length l is 11 feet, width w is 5 feet, and height h is 7 feet.

FOR REVIEW ONLY

Remember to go through all of the problems in the Practice section before attempting to Certify!

1.2.c Properties of Radicals

Simplifying radicals means more than evaluating square roots of numbers. We also want to be able to simplify algebraic expressions that contain radicals. We call these *radical expressions*. There are numerous properties that we'll look at which can be used to simplify these radical expressions.

Learn:

- $\sqrt[n]{a} = b \iff a = \underline{\hspace{2cm}}$
- A radical expression is in **simplified form** when:
 - The contains no factor with an exponent greater than or equal to the index of the radical.
 - The radicand contains no .
 - The denominator, if there is one, contains no .
 - The greatest common factor of the and any exponents occurring in the radicand is 1.
- $\sqrt[n]{a^n} = \begin{cases} |a| & \text{if } n \text{ is } \underline{\hspace{2cm}} \\ a & \text{if } n \text{ is } \underline{\hspace{2cm}}. \end{cases}$
- Complete the following Properties of Radicals.
 - $\sqrt[n]{ab} = \underline{\hspace{2cm}}$.
 - $\sqrt[n]{\frac{a}{b}} = \underline{\hspace{2cm}}$
 - $\sqrt[m]{\sqrt[n]{a}} = \underline{\hspace{2cm}}$
- To simplify the expression $\frac{3 + \sqrt{2}}{5 - \sqrt{3}}$, we would multiply numerator and denominator by the **conjugate** of the denominator. What is the conjugate of the denominator?

Video Examples:

Watch the videos for Section 1.2c on www.hawkestv.com and then solve the following problems.

- Simplify the following radical expression.

$$\sqrt[4]{\frac{y^{12}z^8}{81}}$$

- Simplify the following radical by rationalizing the denominator.

$$\frac{\sqrt{z} + \sqrt{x}}{\sqrt{z} - \sqrt{x}}$$

Practice:

1. Determine if the following radical expression is a real number. If it is, evaluate the expression.

$$\sqrt[3]{-27}$$

2. Simplify the following radical expression.

$$\sqrt[3]{8x^4y^{17}}$$

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3. Simplify the radical expression by rationalizing the denominator.

$$\frac{y}{\sqrt{5x}}$$

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Remember to go through all of the problems in the Practice section before attempting to Certify!

1.2.d Rational Number Exponents

At times, it is useful to represent radical expressions as expressions with rational number exponents. Then, we can use the properties of exponents to simplify these same radical expressions. We'll get some practice doing just that in this section.

Learn:

1. Radical expressions are called **like radicals**, if they have the same _____ and the same _____.
2. Meaning of $a^{1/n}$: If n is a natural number and if $\sqrt[n]{a}$ is a real number, then $a^{1/n} = \underline{\hspace{2cm}}$.
3. Meaning of $a^{m/n}$: If m and n are natural numbers with $n \neq 0$, if m and n have no common factors greater than 1, and if $\sqrt[n]{a}$ is a real number, then $a^{m/n} = \sqrt[n]{\hspace{2cm}} = (\hspace{2cm})^m$.
4. $16^{3/4}$ can be written as $\sqrt[4]{16^3}$ or as $(\sqrt[4]{16})^3$. Which form is easier to calculate? Simplify the expression.

Video Examples:

Watch the videos for Section 1.2d on www.hawkestv.com and then solve the following problems.

- Simplify the following expression.

$$\sqrt[3]{\sqrt[7]{x^{21}}}$$

- Simplify the following expressions.

$$(3x^3 + 2)^{11/3}(3x^3 + 2)^{-10/3}$$

Practice:

1. Simplify the following expression. Assume all variables are positive.

$$(x^{1/2} \cdot y^{-1} \cdot z^{-1/3})^{-2/3}$$

2. Combine the radical expressions, if possible.

$$\sqrt[5]{-729y^6} + 3y\sqrt[5]{3y}$$

3. Simplify the following expression, writing your answer using the same notation as the original expression.

$$\frac{x^{2/3}z^{-1/2}}{x^{-1/3}z}$$

FOR REVIEW ONLY

Remember to go through all of the problems in the Practice section before attempting to Certify!

1.3 Polynomials and Factoring

Polynomials are an important type of algebraic expression and will appear often in this course. With that in mind, let's get some practice in identifying types of polynomials and factoring polynomials.

Learn:

1. Polynomials consisting of a single term are called _____, those consisting of two terms are _____, and those consisting of three terms are _____.

2. What is the degree of the following polynomial? What is the leading coefficient?

$$3x^2 - 7x^3 + 5x + 2$$

Degree:

Leading coefficient:

3. Which terms in the following two polynomials are **like** terms?

$$-2a^3 - 4ab^2 + 7ab + 5b^2$$

$$10ab - 6a^2b + 8b^2 - 2a^2$$

Like terms:

4. What is the greatest common factor in the following polynomial?

$$10a^2b - 6ab + 4a^2b^2$$

GCF:

5. Complete the special binomial factoring formulas.

(a) $A^2 - B^2 = (\quad) (A + B)$

(b) $A^3 - B^3 = (A - B)(\quad)$

(c) $A^3 + B^3 = (\quad) (A^2 - AB + B^2)$

6. Complete the following steps which are used to factor a trinomial $a^2 + bx + c$ where $a \neq 1$.

(a) Multiply a and _____.

(b) Factor ac into two integers whose sum is _____. If no such factors exist, the trinomial is irreducible over the integers.

(c) Rewrite b in the trinomial with the sum found in step 2, and _____.
The resulting polynomial of four terms may now be factored by grouping.

Video Examples:

Watch the videos for Section 1.3 on www.hawkestv.com and then solve the following problems.

- Multiply the following polynomials.

$$(4x^2 + y)(x^2 - 5y)$$

- If possible, factor the following trinomial.

$$12y^2 - 17y + 6$$

Practice:

1. Add or subtract the following polynomials, as indicated.

$$(-7y^3 + 10 - 9y^4) - (2 - 4y^3)$$

2. If possible, factor the following polynomial by factoring out the greatest common factor.

$$15x^2y + 5x^4 - 15x^3y$$

3. If possible, factor the following polynomial by grouping.

$$ax - 4bx + 4ay - 16by$$

Remember to go through all of the problems in the Practice section before attempting to Certify!

1.4 The Complex Number System

In this section, we'll look study *complex numbers*. Just like real numbers, we can add, subtract, multiply, and divide complex numbers. In addition, we will encounter equations in this class which do not have a real number solution, but do have a complex number solution.

Learn:

1. The imaginary unit i is defined as $i = \underline{\hspace{2cm}}$. In other words, i has the property that its square is -1 : $i^2 = \underline{\hspace{2cm}}$.
2. For any two real numbers a and b , the sum $\underline{\hspace{2cm}}$ is a **complex number**.
3. Identify the real part and the complex part of the complex number $-7 + 5i$.
Real part: $\hspace{10em}$ Imaginary part:
4. Determine the complex conjugate of the complex number $3 - 2i$.
Conjugate:
5. What is the product of any complex number $a + bi$ and its conjugate $a - bi$.

$$(a + bi)(a - bi) =$$

Video Examples:

Watch the videos for Section 1.4 on www.hawkestv.com and then solve the following problems.

- Simplify the following square root expression.

$$\sqrt{-20}\sqrt{-2}$$

- Simplify the quotient.

$$\frac{3 - i}{2 + 4i}$$

Practice:

1. Simplify the following expression.

$$(16 - 6i)(10 - 11i)$$

2. Simplify the following expression.

$$i^{43}$$

FOR REVIEW ONLY

3. Simplify the following expression.

$$(6 - 8i)(6 + 8i)$$

FOR REVIEW ONLY

Remember to go through all of the problems in the Practice section before attempting to Certify!

1.5 Linear Equations in One Variable

One of the most basic types of equations are **linear** equations. You probably solve these types of equations often without even writing anything down. For these problems, however, we'll probably need to write some things down. We'll break this section into two parts: solving linear equations and solving application problems involving linear equations.

1.5.a Linear Equations in One Variable

Let's start by getting some practice solving linear equations.

Learn:

1. Is the following equation an **identity**, a **contradiction**, or a **conditional** equation? Can you give an example of each?

$$x + 3 = 5$$

2. A **linear equation in one variable** is an equation that can be transformed into the form _____, where a and b are real numbers and $a \neq 0$.
3. To solve an equation with fractions, we can get rid of the fractions by multiplying each side of the equation by the least common _____ or LCD. To get rid of the fractions in the equation below, by what number should each side of the equation be multiplied?

$$\frac{x}{3} + \frac{x-2}{6} = \frac{x+5}{2} \quad \text{LCD} = \underline{\hspace{2cm}}$$

4. What two values for x would make the following equation true?

$$|x| = 6$$

$$x = \underline{\hspace{2cm}} \text{ and } x = \underline{\hspace{2cm}}$$

5. Without solving, how many solutions would the following equation have? Why?

$$|x - 2| + 10 = 0$$

Video Examples:

Watch the videos for Section 1.5a on www.hawkestv.com and then solve the following problems.

- $0.5t + 1.4 = 3t$

- $|3x - 7| = 13$

Practice:

1. Solve the following linear equation.

$$2t + 1 = 3(t + 2) - 5$$

FOR REVIEW ONLY

2. Solve the following linear equation. Remember it may be easier to multiply each side of the equation by the LCD first.

$$\frac{2y + 3}{4} + \frac{3}{2} = \frac{3y - 1}{2}$$

3. Solve the following absolute value equation. Don't forget to check your answers.

$$|x + 5| = |x - 4|$$

Now complete the rest of the problems in the Practice section on your Hawkes courseware and then you'll be ready to Certify!

1.5.b Applications of Linear Equations in One Variable

What good is learning all of this mathematics if we can't apply it? In this section, we'll look at some application problems that can be solved with linear equations.

Learn:

1. Two equations that have the same solution set are called _____ equations.

2. **Solving for a variable** means to transform the equation into an equivalent one in which the specified variable is _____ on one side of the equation.

3. Write the basic distance formula.

4. In the simple interest formula $I = Prt$, what does the variable P represent?

Video Examples:

Watch the videos for Section 1.5b on www.hawkestv.com and then solve the following problems.

- Find three consecutive odd integers whose sum is 189.

- Suppose two ships leave a port at the same time. The first travels east at 24 miles per hour and the second travels west at 30 miles per hour. When will the ships be 81 miles apart?

Practice:

1. Solve the following formula for the indicated variable.

$$F = \frac{9}{5}C + 32$$

2. Suppose your bill for a meal comes to \$18.76. If you want to leave an 18% tip, how much should you pay?

3. If \$5000 is invested in a savings account which pays 2.4% annually, how much money will be in the account after 1 year?

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Now complete the rest of the problems in the Practice section on your Hawkes courseware and then you'll be ready to Certify!

1.6 Linear Inequalities in One Variable

Oftentimes, we want to find all of the numbers that make a particular inequality true. Since there will usually be infinitely many solutions to such a problem, we'll need to represent our answers either graphically or using interval notation.

Learn:

1. Fill in the blanks with $<$ or $>$.

(a) If $C > 0$, $A < B \iff A \cdot C \underline{\hspace{1cm}} B \cdot C$.

(b) If $C < 0$, $A < B \iff A \cdot C \underline{\hspace{1cm}} B \cdot C$.

2. If the solution to a linear inequality is $x < 5$, graph this solution and write it using interval notation.

3. A **compound inequality** is a statement containing two inequality symbols, and can be interpreted as two distinct inequalities joined by the word “_____”.

4. To solve the absolute value inequality $|2x + 6| > 3$, what two independent inequalities would you solve?

or

Video Examples:

Watch the videos for Section 1.6 on www.hawkestv.com and then solve the following problems. Write your solutions using interval notation.

- $2|a + 4| \leq 16$

- $-3(x + 2) \leq 12$ or $15 + x < 17$

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Practice:

1. Solve the following compound inequality.

$$\frac{6}{8} < \frac{y + 2}{4} < \frac{18}{8}$$

2. Solve the following absolute value inequality.

$$4|2 - r| \leq 8$$

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3. In a class in which the final course grade depends entirely on the average of four equally weighted 100-point tests, Jason has scored 87, 91, and 83 on the first three. What range of scores on the fourth test will give Jason a B for the semester (an average between 80 and 89, inclusive)? Assume that all test scores have a non-negative integer value.

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Now complete the rest of the problems in the Practice section on your Hawkes courseware and then you'll be ready to Certify!

1.7 Quadratic Equations

Of course, many of the equations that you will be asked to solve in this class aren't going to be linear. When the highest power on the variable in an equation is 2 or bigger, you will have to use different techniques to solve these types of equations. This section looks at some of those methods used to solve these higher degree equations.

1.7.a Quadratic Equations in One Variable

One of the most common types of equation is the quadratic equation. There are numerous methods that can be used to solve quadratic equations. Determining which method is best for a particular equation is part of what you'll learn in this section.

Learn:

1. A **quadratic equation in one variable**, say the variable x , is an equation that can be transformed into the form _____, where a, b , and c are real numbers and $a \neq 0$.
2. If A and B are algebraic expressions and $AB = 0$, then _____ = 0 or _____ = 0.
3. If you wanted to solve the equation $x^2 + 6x - 4 = 0$ by **completing the square**, what would the equation look like after the first step?

4. According to the **quadratic formula**, the solutions of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5. If a ball is thrown upward with a velocity of 20 ft/s from a height of 42 ft, what values would be substituted for v_0 and h_0 in the formula $h = -\frac{1}{2}gt^2 + v_0t + h_0$?

$$v_0 = \text{_____}, h_0 = \text{_____}$$

Video Examples:

Watch the videos for Section 1.7a on www.hawkestv.com and then solve the following problems.

- Solve the equation $x^2 + 6x + 9 = 64$ by the square root method.

- Solve the equation $x^2 + 8x + 7 = -8$ by completing the square.

FOR REVIEW ONLY

Practice:

1. Solve the following quadratic equation by factoring.

$$y^2 + 3y - 28 = 0$$

2. Solve the following quadratic equation using the quadratic formula.

$$2x^2 - 4x - 3 = 0$$

FOR REVIEW ONLY

3. How long would it take for a stone dropped from the top of a 196-foot building to hit the ground?

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Now complete the rest of the problems in the Practice section on your Hawkes courseware and then you'll be ready to Certify!

1.7.b Higher Degree Polynomial Equations

Of course there are numerous equations with a degree of more than 2 that can be solved. In this section, we will look at some of the methods that are used to solve these higher degree polynomial equations.

Learn:

1. An equation is quadratic-like, or quadratic in form, if it can be written in the form _____ where a, b , and c are constants, $a \neq 0$, and A is an algebraic expression.
2. In the quadratic-like equation below, determine the value of A so that the equation can be written as $aA^2 + bA + c = 0$.

$$(x^3 + x)^2 - (x^3 + x) - 12 = 0 \quad A = \underline{\hspace{2cm}}$$

3. To solve $8x^3 - 27 = 0$ by factoring, we would write the left side of the equation as a difference of two cubes. What are the two things being cubed?

$$(\quad)^3 - (\quad)^3 = 0$$

4. In the equation $x^{5/2} + 2x^{3/2} - 8x^{1/2} = 0$, what power of x can be factored out on the left side of the equation?

Video Examples:

Watch the videos for Section 1.7b on www.hawkestv.com and then solve the following problems.

- Solve the following equation by grouping.

$$2x^3 + x^2 + 2x + 1 = 0$$

- Solve the following equation.

$$3x^{13/5} + 2x^{8/5} - 5x^{3/5} = 0$$

Practice:

1. Solve the following polynomial equation.

$$2y^3 + 2y^2 = 4y$$

2. Solve the following quadratic-like equation.

$$(z^2 - 7)^2 + 7(z^2 - 7) - 18 = 0$$

3. Solve the following polynomial equation.

$$8x^3 - 1 = 0$$

FOR REVIEW ONLY

Now complete the rest of the problems in the Practice section on your Hawkes courseware and then you'll be ready to Certify!

1.8 Rational and Radical Equations

Two other types of equations that you will encounter are *rational equations* and *radical equations*. Each type of equation has its own methodology for finding a solution.

1.8.a Rational Expressions and Equations

Fractions that involve algebraic expressions are called rational expressions. In this section, we will look at ways to combine and simplify these expressions as well as solving equations involving these expressions.

Learn:

1. A **rational expression** is an expression that can be written as a _____ of two polynomials $\frac{P}{Q}$ (with $Q \neq 0$).
2. In the following rational expression, how can we rewrite the denominator to cancel a common factor?

$$\frac{x^2 - 3x - 10}{5 - x} = \frac{(x + 2)(x - 5)}{-\left(\quad\right)}$$

3. In order to add or subtract two rational expressions, we must first find the _____ or LCD.
4. In the following complex rational expression, what is the LCD of all the fractions?

$$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y^2}} \quad \text{LCD} = \underline{\hspace{2cm}}$$

5. In the following rational equation, what is the LCD that should be multiplied by both sides of the equation?

$$\frac{x}{x-1} + \frac{2}{x-3} = \frac{2}{x^2 - 4x + 3} \quad \text{LCD} = \underline{\hspace{2cm}}$$

Video Examples:

Watch the videos for Section 1.8a on www.hawkestv.com and then solve the following problems.

- Simplify the following rational expression.

$$\frac{x^2 + 2}{x - 3} - \frac{x + 4}{x + 5}$$

- Solve the following rational equation.

$$\frac{2}{2x + 1} - \frac{x}{x - 4} = \frac{-3x^2 + x - 4}{2x^2 - 7x - 4}$$

Practice:

1. Add or subtract the following rational expressions.

$$\frac{x+3}{x-3} - \frac{x-1}{x+6} - \frac{2}{x^2+3x-18}$$

2. Simplify the following complex rational expression.

$$\frac{8y+32}{2 - \frac{32}{y^2}}$$

3. If Kendra were to paint her clean her bathroom alone, it would take 2 hours. Her sister Kendra could do the job in 4 hours. How long would it take them working together?

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Remember to go through all of the problems in the Practice section before attempting to Certify!

1.8.b Radical Equations

Some equations are a little more out there than others. These are called radical equations and there is a very specific set of steps we can use to solve them, which we'll look at in this section.

Learn:

1. Fill in the blanks for the Method for Solving Radical equations.

Step 1: Begin by _____ the radical expression on one side of the equation. If there is more than one radical expression, choose one to isolate on one side.

Step 2: Raise both side of the equation by the power necessary to “_____” the isolated radical. That is, if the radical is an n^{th} root, raise both sides to the _____ power.

Step 3: If any radical expression remain, simplify the equation if possible and then repeat steps 1 and 2 until the result is a _____ equation. When a polynomial equation has been obtained, _____ the equation using polynomial methods.

Step 4: _____ your solutions in the original equation! Any extraneous solutions must be discarded.

2. Isolate the radical in the following equation.

$$\sqrt{x-3} + 4 = x$$

$$\sqrt{x-3} =$$

3. To what power should we raise both sides of the following equation?

$$(2x^2 - 4x + 12)^{1/5} = 3 \quad \text{______}^{\text{th}} \text{ power}$$

Video Examples:

Watch the videos for Section 1.8b on www.hawkestv.com and then solve the following problems.

- Solve the following radical equation.

$$\sqrt{50 + 7s} - s = 8$$

- Solve the following equation.

$$x^{3/5} - 8 = 0$$

Practice:

1. Solve the following radical equation.

$$\sqrt{7x + 11} + 6 = x + 5$$

2. Solve the following radical equation.

$$\sqrt[3]{6y^2 + 27y} = \sqrt[3]{2y^2 + 40}$$

3. Solve the following equation.

$$z^{2/3} - \frac{49}{64} = 0$$

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Now complete the rest of the problems in the Practice section on your Hawkes courseware and then you'll be ready to Certify!